

PROMOTING ENTRY IN TELECOM AUCTIONS

A thesis presented

by

Osnat Yaari-Tikochinsky

as a requirement for the degree of

Doctor of Philosophy (PhD)

to

The Department of Economics

University College London

July 2005

UMI Number: U602688

All rights reserved

INFORMATION TO ALL USERS

The quality of this reproduction is dependent upon the quality of the copy submitted.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if material had to be removed, a note will indicate the deletion.



UMI U602688

Published by ProQuest LLC 2014. Copyright in the Dissertation held by the Author.
Microform Edition © ProQuest LLC.

All rights reserved. This work is protected against
unauthorized copying under Title 17, United States Code.



ProQuest LLC
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106-1346

Abstract

This study is situated at the junction between the theoretical and the practical aspects of auction theory. We are particularly interested in the question of participation (or entry) of bidders in auctions. This is essentially a theoretical question, however the motivation for studying it stems from the observation of real auction situations. In the case of high-profile telecom auctions - auctions of licences for operating telecommunications technology - the problem of bidder participation poses a particularly acute challenge, that needs to be addressed in the design of the auction.

The thesis contains five chapters; first an introduction that follows the appearance of the entry-problem in telecom auctions, and the motivation for studying it is provided (chapter 1), then the related theoretical literature is surveyed (chapter 2). The main body of the thesis is brought in chapters 3,4 and 5.

Chapter 3 develops a theoretical model in which the use of royalty-bidding in telecom auctions is studied. The aim of capturing the main aspects of a real-life telecom auction environment, imposes a certain structure on the task of modelling it for the purpose of theoretical analysis. Our main interest is to examine whether the entry of weak bidders (in telecom auctions these are usually newcomers), could be promoted by the introduction of royalties to the auction, and to study the effects of

such actions on other participants in the market. For that purpose a 'second-price royalty auction with a fixed-fee element' is modelled.

Chapter 4 expands the analysis of chapter 3 by changing the basic setting of the model. In particular, the assumptions at the basis of the model, regarding the knowledge at the auctioneer's disposal when she designs the auction, are made less restrictive.

In chapter 5 we abandon the use of royalties in auctions altogether, and turn to an experimental examination of the question of entry within two of the most common auction formats in the telecom industry - the ascending price auction and the first-price sealed bid auction. Our aim in this chapter is to compare the entry-promoting properties of the two auctions by observing the entry behaviour of bidders in a monitored environment where the only variable is the auction format.

Contents

1	Introduction and Motivation	14
1.1	The Telecom Auctions Experience	15
1.2	The Entry Issue	22
1.2.1	What Do We Mean by ‘Entry’?	23
1.2.2	Why is Entry Important?	25
1.2.3	The Appearance of the Entry Problem	27
1.2.4	Ways Used to Promote Entry via the Auction Design	28
1.2.5	Creating an ‘Entry-Friendly’ Telecom Market	30
1.3	The Use of Royalties and Royalty-Bidding	31
1.3.1	Basic Underlying Assumptions	32
1.3.2	How Would a Risk-Averse Bidder Behave?	34
1.3.3	The Use of Royalty-Bidding	37
1.3.4	The Second-Price Royalty Auction with a Fixed-Fee Element	38
2	Theoretical Literature Survey	42
2.1	Auctions with Entry	43
2.2	Auctions with Risk-Averse Bidders	47
2.3	Auctions with Asymmetric Bidders	52
2.4	Auctions where the Winners’ Payment is not a Function of Bids Alone	57

3 The Second-Price Royalty Auction with a Fixed-Fee	
Element the ‘one incumbent - one entrant’ case	66
3.1 Description of the Game	67
3.1.1 The Market	67
3.1.2 The Auction.....	69
3.1.3 The Entry Stage	71
3.2 Analysis of the Three-Stage Game	72
3.2.1 The Market	72
3.2.2 The Auction.....	74
3.2.3 The Entry Stage	77
3.3 Do Royalties Promote Entry?	85
3.3.1 The Market	86
3.3.2 The Auction.....	87
3.3.3 The Entry Stage	89
3.4 Revenue and Entry	91
3.4.1 The Auctioneer’s Revenue	93
3.4.2 Revenue and Entry - a trade-off	97
3.4.3 The Optimal Fixed Fee	100
 4 The Second-Price Royalty Auction with a Fixed-Fee	
Element the case of multiple incumbent’s types	103
4.1 Description of the New Model	104
4.1.1 The Market	105
4.1.2 The Auction.....	106

4.1.3	The Entry Stage	106
4.2	The Auctioneer's Task	111
4.2.1	The Auctioneer's Revenue	112
4.2.2	Entry Takes Priority Over Revenue	118
4.2.3	Constructing the Optimal $F(r)$ Point-By-Point	118
4.2.4	Are We On The Equilibrium Path?	122
4.2.5	The Final Shape of the Optimal Fixed-Fee Schedule	134
4.3	The Hong Kong Royalty Auction	135
4.3.1	The Basic HK Auction Design	136
4.3.2	A Simple Analysis of the HK Auction Environment	137
5	Entry Into Auctions - An Experimental Approach¹	146
5.1	Vickrey's Example	149
5.2	Experimental Setting	150
5.2.1	Equilibrium Analysis	154
5.3	Results	162
5.3.1	Bidding Behaviour	164
5.3.2	Entry Behaviour	169
5.3.3	Auctioneer's Revenue	184
5.4	Conclusion	187
5.5	Related Literature	188
A	A possibility for a loss in the telecom market	196

¹ This chapter is based on joint work with Joe Swierzbinski and Chris Tomlinson.

A.0.1 The Market	197
A.0.2 The Auction.....	198
Bibliography	204

List of Figures

• Figure 1: The regular ‘cash’ second-price auction.....	34
• Figure 2: The ‘pure’ royalty auction.....	35
• Figure 3: E_{tie} is increasing with I	79
• Figure 4: E_{tie} is decreasing with F	80
• Figure 5: Different fixed-fee schedules.....	82
• Figure 6: The threshold royalty rate r^* drops as F increases.....	85
• Figure 7: The incumbent’s bid ρ^I drops as F increases.....	96
• Figure 8a: Auctioneer’s revenue and entry curves.....	98
• Figure 8b: The welfare function.....	99
• Figure 9a: Several fixed-fee schedules with $I = 0$	108
• Figure 9b: Several fixed-fee schedules with $I > 0$	108
• Figure 10: Entry with different fixed-fee schedules.....	109
• Figure 11: The entrant’s disadvantage for different $F(r)$	110
• Figure 12a: The incumbent’s bid for given I and F	115

• Figure 12b: The incumbent's bid for different fixed-fee schedules.....	116
• Figure 13a: The optimal F is derived from the incumbent's payoff function.....	124
• Figure 13b: The optimal F is unique.....	124
• Figure 14: The optimal fixed-fee schedule.....	127
• Figure 15: What bidders know at the beginning of the game.....	129
• Figure 16: The optimal F is flatter the more risk averse is the entrant.....	131
• Figure 17: The optimal F is flatter the larger is $(I_N - I_0)$	133
• Figure 18: The optimal fixed-fee schedule.....	134
• Figure 19: The Hong Kong royalty auction.....	140
• Figure 20: Examples of different fixed-fee schedules for HK.....	142
• Figure 21: A sketch of the HK fixed-fee schedule.....	142
• Figure 22a: The information in the HK bidders' disposal.....	143
• Figure 22b: The HK bidders' profit curve.....	143
• Figure 23a: Underdog's bid function.....	160
• Figure 23b: Topcat's bid c.d.f.....	160

- **Figure 24:** Losing bids in the ascending price auction.....165
- **Figure 25:** Empirical and equilibrium c.d.fs for Topcat's bids.....167
- **Figure 26:** Underdog's observed and equilibrium bids.....168
- **Figure 27:** Ascending auction's clearing price as a function of Underdogs
valuation.....186
- **Figure 28:** Descending auction's clearing price as a function of Underdogs
valuation.....186

List of Tables

- **Table 1:** The appearance of the ‘entry problem’ in telecom auctions in Europe.....27
- **Table 2:** Percentage of bidders entering in each auction format.....170
- **Tables 3a-3e:** A bidder-by-bidder analysis of entry behaviour of Underdogs in the ascending price treatment.....171-173
- **Tables 4a-4e:** A bidder-by-bidder analysis of entry behaviour of Underdogs in the descending price treatment.....173-175
- **Tables 5a-5e:** A bidder-by-bidder analysis of entry behaviour of Topcats in the ascending price treatment.....177-179
- **Tables 6a-6e:** A bidder-by-bidder analysis of entry behaviour of Topcats in the descending price treatment.....180-182

Acknowledgments

First and foremost I would like to thank my primary supervisor Ken Binmore, for his incredible generosity with his time and effort. His guidance and support throughout my work on this project made it possible to regroup and continue even when it seemed fruitless and when, on countless occasions, a change of direction was required. His genuine care for his students made this long journey seem less lonely. I feel truly fortunate for the opportunity I had to work with such an inspirational man.

I would also like to thank my secondary supervisor Philippe Jehiel, and Richard Vaughn for comments on an early version.

Special thanks to Joe Swierzbinski with whom I worked on the auction experiment. I learnt so much from his vast knowledge and experience in both theory and experimental work. I am sure this asset will serve me in future research projects. And to Chris Tomlinson who wrote the computer programme for the experiment, maintained and changed it countless of times, according to our changing needs. Thanks to Tom Rutter who runs the computer lab and summons the subjects.

I thank my co-students Martin Boeg, Fabio Michelucci, and Pedro Rey Biel, with whom I share an office and who provided a very pleasant and friendly working environment.

I would like to thank my family, and in particular my brother Jonathan Yaari for computer support, my mother Malka Yaari for countless hours of baby-sitting that

allowed me uninterrupted time to work. Special thanks to my children Shira and Nadav who had to make do with less than a full-time-mum.

And last but not least I thank my husband Jonathan Tikochinsky who believed in me even when I had doubts, and supported me in every possible way. This project would not have been able to reach its finishing line without him.

Chapter 1

Introduction and Motivation

Auctions have been used, in various shapes and forms, as sale mechanisms in the presence of asymmetric information, since ancient times. Popular examples include the sale of wives on the streets of Babylon, (Herodotus), and the sale of the Roman Empire to Didius Julianus, (193AD), both of which are thought to have been conducted by some early form of auction. It was however only with Vickrey's pioneering work in the 1960s that theory finally caught up with practice. Many models have now been developed to study and analyse these widely used market mechanisms. Auctions have increasingly become one of the major areas in which micro-economic theory can actually be put to work in practice. They present an opportunity for creating models that genuinely capture aspects of specific real world situations.

It is at this junction between the theoretical and the practical that this study is situated. We shall be particularly interested in the question of participation (or entry) of bidders in auctions. This question is essentially a theoretical one, however the motivation for studying it stems from the observation of real auction situations. In the case of high-profile telecom auctions - auctions of licences for operating telecommu-

nications technology – the problem of bidder participation poses a particularly acute challenge, that needs to be addressed in the design of the auction.

The aim of capturing the main aspects of a real-life telecom auction environment, imposes a certain structure on the task of modelling it for the purpose of analysis. There is a cost and a benefit to tying the theory and the practice together; a trade-off exists between developing a general, widely applicable tool and aiming at a more specific analysis, better fitted to a certain situation as it is observed ‘on the ground’. The presence of this trade-off will be felt throughout this study, where we go back and forth between the theory and the real world.

In the remainder of this chapter we will describe the practical background and formation of the ‘entry problem’ in the framework of telecom auctions experience. The next chapter will present the theoretic literature where certain aspects of the question posed here have been addressed.

1.1 The Telecom Auctions Experience

The allocation of licences that entitle their owners to use a segment of the radio spectrum for commercial purposes, is a natural candidate for the use of auctions. There is a scarce, government-owned resource on the one side and a handful of buyers, whose willingness to pay is unknown to the government, on the other.

The idea was discussed in the US Congress as early as 1958, but it was only several decades later that the first telecom auctions took place. During this time govern-

ments used a variety of bureaucratic processes for allocating bands of radio spectrum to companies claiming to be in the best position to operate adequate telecommunications services. These processes were given the collective name ‘beauty contests’, perhaps to hint at their subjective nature. It was also a very long and non-transparent process as it is very difficult to define a specific set of criteria for selecting the best of the candidates and to justify this process, if required, in retrospect. Governments could find themselves exposed to allegations of corruption and favouritism that could be difficult to shake. Such problems with the beauty contests led to the US Congress replacing them, in the 1980s, with lotteries, in the hope of streamlining the process.

Lotteries were indeed a simple and transparent way for allocating, on a random basis, a large number of licences among the vast number of companies that applied to participate. The problem with lotteries is that an efficient allocation is as likely to emerge as any other specific allocation. In fact, both beauty contests and lotteries sometimes resulted in allocations that were widely thought to be inefficient. Indeed, sizable secondary markets developed in which winners of licenses, who could not, or never had any intention of, supplying customers with the relevant services, would sell their licence to companies better suited for the task, while making huge profits in the process. The idea that auctions might provide a way out of this mess, therefore gradually gathered support.

The first telecom auction took place in New Zealand in 1990, where the government chose the second-price, sealed-bid auction design to allocate spectrum for

radio, television, and cellular telephones. Next was the Australian government, who chose the first-price, sealed-bid design for their telecom auction, run in 1993. Neither of these auctions was viewed as successful, as design inadequacies - most obvious of which were the lack of a meaningful reservation price in the New Zealand case and the absence of deposits or other penalties for bid withdrawals in the Australian case - left them vulnerable to bidder manipulation and public criticism.

Auctions eventually achieved their current reputation as a simple and efficient tool for selling telecom licences, only after the American Federal Communications Commission (FCC) adopted the sealed-bid, multi-round, simultaneous design in 1994, as advocated by economic theorists, and began using auctions for allocating licences to hundreds of companies across the US. This design also served as a foundation for many of the later European telecom auctions.

Before we take a brief look at the sealed-bid, multi-round, simultaneous auction design, it is useful to recall some of the objectives of the auction, as stated by the US Congress:

- winners should use the spectrum in an efficient and intensive manner
- rapid deployment of technologies should be promoted
- excessive concentration of licenses should be prevented
- some minority-owned and other designated bidders should be ensured a license.

(See McMillan 1994 for a detailed account of the aims and the process of the auction design.)

It is interesting to note at this point that maximising revenue did not enter this list as a separate goal, (although it is folded into the efficiency objective to some extent). On the other hand, the 'spirit' of promoting entry into the auction is already present here, both in the wish to prevent concentration of licences (for which purpose a large number of bidders is desirable), and in the identification of some potential bidders as "designated", with its implicit assumption that some bidders stand on a disadvantageous ground with respect to others.

The FCC auction was the first in which the sealed-bid multi-round, simultaneous auction design had been used. The team of economic consultants who were involved in the design combined elements of several well-known auction formats, as well as inventing brand new ideas, to create this new and vast sale mechanism. Naturally this auction had a large number of rules ensuring its smooth running. For the purpose of this study, we will only state some of them briefly, in order to create a basis for comparison between telecom auctions. (For more on the FCC auction design see, for example, McAfee & McMillan 1996, and Cramton 2001).

In each of the FCC auctions a large number of licences were put on sale. Each licence corresponded to a fraction of the radio spectrum, and its winner held the right to use these air waves in order to provide some specified telecommunications services to consumers, in a certain geographic area. Firms who qualified to participate could

bid for any of the licences offered. A bid that is put on a licence is a commitment to buy that licence at the price specified in the bid, if no other bidder is willing to pay more for it.

The 'multi-round sealed bid' aspect of the auction could be thought of as the discrete-time equivalent to the English, or open, auction. Its operative meaning is that bidders actually submit a sealed bid in each round, and when every bidder has submitted a single bid (which doesn't necessarily apply just to a single licence) a round is completed. At the end of each round the results are announced before the next round begins. The results consist of the bids and identities of the bidders who submitted the highest bids. Based on this information bidders prepare their next round's bids which, again, are submitted sealed. The auction is over when no bidder wants to submit a new bid on any of the licences.

This design captures some of the advantages of an open auction; in particular, it allows information about the bidders' valuations for the licences to be revealed during the course of the bidding. This feature adds to the transparency of the auction, and to the simplicity of the bidding process, but it can be of particular importance in telecom auctions where both the collusion among bidders, and cautious bidding due to the effect of the winner's curse are possibilities that the auctioneer needs to take into account. The licences being auctioned have a significant common-value aspect to them, in that the variation in the valuation of each licence among different bidders derives, at least in part, from different estimations of features of the licence

which are the same for all bidders, and to which they would therefore assign the same value if they were fully informed. Commonly-valued goods are more vulnerable to the effects of the 'winner's curse' which when properly taken into account in the bidding, leads bidders to bid cautiously. In the framework of an open auction this means that each time a bidder makes a new bid that outbids some of the opponents, he/she has to take into account the information that is revealed by the mere fact that other bidders were not willing to bid this high, on the estimated value of the licence. This information has a negative effect on the licence's value, and thus a bidder who anticipates this event will lower his/her bids in advance. On the other hand, the fact that bids were submitted sealed allows the auctioneer to control the release of information, as bidders do not know who is competing directly against them, for a particular licence in each round, until the round is over. The control over the release of some bidding information is also important for reducing, to some degree, the chances of collusion among bidders during the auction.

The 'simultaneous' in the description of the auction refers to the fact that all licences are sold at the same time to allow for better licence aggregation. This is important in telecom auctions as different licences can serve as either substitutes or complements to one another in the eyes of different bidders. The government usually has no way of bundling the licences efficiently before the auction.

The FCC's auctions were generally viewed as successful. Indeed a decade later, although some mistakes have been identified and changes introduced, the same basic

design is still used in telecom auctions. Moreover, the FCC auction design served as a basis for the design of most of the European and other telecom auctions that took place later.

We now turn to look briefly at the telecom auctions experience in other parts of the world. The UK was the first among the European countries to use an auction for assigning licences for operating third generation (3G) telecommunications technology. The UK auction took place in the spring of 2000, and allocated 5 such licences among 13 bidders. Of these bidders 4 were the incumbents in the British telecom market, and 9 were new entrants. The design of the UK auction was based on that of the FCC, adapted to fit the British market. A detailed account of the design and the conduct of the UK auction can be found in Binmore & Klemperer 2002. For the purpose of this study, it is sufficient to note that the 5 licences were sold using a simultaneous ascending-price auction, in which bidders were allowed to bid on a single licence in each round. The bidding on the largest of the licences was restricted to new entrants, while all bidders were allowed to bid for the other 4. The British auction is viewed as a success in most respects, and in particular it generated a huge amount of revenue for the government, (GBP 22.47 billion), that exceeded all expectations. It was regarded at the time as ‘the biggest auction ever’.

Other European countries followed closely after the UK in allocating 3G licences. Different countries faced different conditions in telecom markets at the time of the auction; moreover, different governments had different aims and objectives

for the process of allocating the licences. A variety of auction designs and beauty contests were used, and thus the results, both in terms of allocations and in terms of revenues, varied a great deal. (See Jehiel & Moldovanu, 2001 or Borghers & Dustmann, 2003 for a detailed description and discussion of 3G licensing across Europe).

It is difficult to talk about the results of different telecom auctions in terms of success, as different auctions are set out to achieve different goals. We would like however, to look at an isolated specific aspect of these auctions, and study their designs on the basis of that. This aspect is bidder participation, or entry.

1.2 The Entry Issue

The great strength of auctions, in comparison to beauty contests or lotteries, is also what makes them less ‘friendly’ to small businesses and new-comers. Auctions require that interested potential buyers be willing to back their business plans with real money bids, as every bid made in the auction is a commitment on behalf of the bidder to pay the amount specified in the bid if no other bidder outbids it, whereas bureaucratic processes often judge participants on their plans alone. It is therefore of paramount importance that bidders should have a good as possible estimate of how much the goods on sale are worth to them, or they could find themselves committed to pay more than is worthwhile, or even more than they can. In the case of telecom auctions such an estimate, especially when the market is essentially new (as with 3G), could be very costly to obtain. These costs that bidders have to incur be-

fore they can bid in the auction effectively serve as entry costs, although they are not paid to the auctioneer, and they could deter the weaker bidders from even trying to compete.

Who are these weaker bidders? Starting a business in the telecom market is a highly risky and costly process. It involves investing the high costs of building a network and establishing a consumer base, before any income can be generated. These costs may be significantly lower for the incumbents in the market, which would give them an advantage in the bidding competition. The combination of a substantial entry cost with a disadvantageous position in the bidding, makes the auction seem like an unattractive business opportunity in the eyes of new potential entrants. The participation of these entrants in telecom auctions may therefore have to be promoted through the auction design. In what follows we present a more thorough account of the entry problem in telecom auctions.

1.2.1 What Do We Mean by ‘Entry’?

It is first necessary to identify precisely what is meant by the term ‘entry’ in our context, and to distinguish it from the more common use of the word. Entry refers to the number of bidders participating in the auction, with a focus on the bidders in the auction who are not incumbents already operating in the relevant telecom market. This use of the term should not be confused with entry to the telecom market itself, with the possible displacement of an incumbent after the auction. The two uses of

the term, although indeed connected, (it may even be argued that the second stems from the first) portray two different concepts, and present different challenges to the auction design.

Consider the following example of an auction, where many bidders come forward to participate, some of whom are new entrants, but where at the end of the bidding, all licences turn out to be bought by incumbents. This auction scores high on entry of the first kind, as many bidders did participate, but it performs poorly on entry of the second kind, as competition in the resulting telecom market remains the same after the auction. Our concern in this study is with the former concept – entry to the auction.

For an incumbent firm in the telecom market, i.e. a firm already selling some telecommunications services to customers, acquiring the new licence now on sale may be the only way for maintaining its competitive position in the market. It may also find it considerably cheaper to obtain their valuation estimates for the new product from their established position within the market, which together with the advantages of having the existing network and consumer base to fall back on, can lower the cost of entry to the auction considerably. It is therefore assumed that all incumbents will choose to enter and compete in the auction. (this assumption is supported by the evidence from telecom auctions around the world). An entrant is therefore a firm, new to the relevant telecom market, who chooses to bid in the auction.

1.2.2 Why is Entry Important?

Assume for the moment that there are 4 incumbents in the telecom market of a certain country, supplying second generation services to customers. Now this country is offering to auction 4 licences to operate third generation technology. The 4 incumbents are the only ones to come forward and bid in the auction, and the licences are therefore sold to them for their reservation prices. Why should the auctioneer wish to prevent this scenario from happening? The short answer is that what was described above is not really an auction. All the advantages that auctions possess over beauty contests and lotteries fail to materialise in the scenario described above.

Even if the current structure of the telecom market with 4 incumbents is quite competitive, and the government does not necessarily want to introduce a fifth operator to the market, it is still non-trivial to argue that the incumbents, rather than some other potential operator, are best suited for the job of operating the new licences, indeed there are examples of cases where an incumbent lost the bidding to a new entrant (e.g. in Denmark). A real bidding competition, in the process of an auction, where the number of bidders is significantly larger than the number of licences, would select the 4 most efficient firms to operate the new technology, and will award them the licences. That is the 'job' of the auction, and it cannot be done without an active bidding competition.

Moreover, the true value of the licences is unknown to the government. This is the source of the asymmetric information and one of the reasons for using an auction

in the first place. The government would therefore have a difficult time setting a realistic reservation price that truly reflects the market value of the licences. This again is the role of the auction – to extract private information from bidders in order to determine how much the licences are worth to each of them, and set the prices accordingly. With no bidding competition the auction cannot be used to determine prices, which would then have to be set by the auctioneer, and could be perceived in this framework as arbitrary. It is exactly this perception of arbitrariness, or worse of favouritism, that governments are trying to avoid by switching from beauty contests to auctions. An active bidding competition is a transparent process which is hard to manipulate, and could serve as a tool in the government's hands for shaking off any allegations of misconduct that may arise after the auction (beauty contests are known to suffer from such allegations, which may some times be hard to prove as unfounded).

It is important to note at this point that, considering the scenario above, it may still be the case that a real and active bidding competition takes place at the end of which the 4 incumbents still win a licence each. The fact that all incumbents, or even that only incumbents, win a licence does not, on its own, mean that the auction was inefficient or unsuccessful. It may be the case that the experience and cost advantages held by incumbents make them the most efficient among the bidders, and that since the market is too narrow to accommodate an additional operator, it is in fact efficient that only incumbents win a licence. The incumbents should, however, be required to

‘prove’ their superiority by backing it with willingness to pay, which is exactly what the auction requires them to do.

1.2.3 The Appearance of the Entry Problem

In the British 3G auction of spring 2000, where 13 bidders competed for 5 licences in a market of 4 incumbents, there was no sign of a problem in attracting bidders to participate in the auction. Nevertheless the designers of the UK auction did see as one of their concerns to ensure that sufficient entry will occur. Indeed the problem did not take long to reveal itself, and in the European telecom auctions that followed the number of new entrants declined steadily. Table 1 below demonstrates the appearance of the ‘entry problem’ in 3G auctions in Europe (information from Borgers and Dustmann 2003, and Jehiel and Moldovanu 2001).

Country	No. of incumbents	No. of bidders at the beginning of the auction	No. of licences on sale
UK	4	13	5
Netherlands	5	6	5
Germany	4	7	6
Italy	4	6	5
Austria	4	6	6
Belgium	3	3	4
Greece	3	4	3
Denmark	4	5	4

Table 1: The appearance of the ‘entry problem’ in Europe.

It is clear from the information² in table 1 that bidder participation did present a problem in many of the European 3G telecom auctions. In some of the cases, however, the issue of entry was addressed in the design of the auction. It is interesting to note those aspects of the auction design specifically introduced to promote entry.

1.2.4 Ways Used to Promote Entry via the Auction Design

The first country to build entry into the design of the auction was the US, where in the early auctions, bidders who were believed to be weaker were titled ‘designated bidders’ and given credits in the bidding process. These credits took the form of a percentage of their bids, such that designated bidders could win a licence as long as their bids lie within that percentage of the highest bid at the final round. A different method was also tried in the early auctions in the US, but was later abandoned – a generous installments payment programme according to which designated bidders had to come up with only a very small fraction of the price at the time of the awarding of the licence. This scheme was cancelled since it attracted speculative bidding in which bidders with no genuine intention of providing a service bought licences in the hope of picking up a quick profit in the resale market.

It is useful to note that, considering the vast telecom market in the US and the large number and types of licences auctioned (most offering regional rather than

² The number of licences in Germany and Austria was variable between 4-6, and determined in the process of the auction.

national coverage), it is likely that the US Congress was concerned more with the second type of entry mentioned above – entry to the market – when it instructed the FCC to promote the entry of designated bidders. Indeed, Congress did specify that the FCC is to ensure that some designated bidders are awarded a licence, rather than simply participate in the auction. It is however, difficult to guess the amount of competition the US auctions would have drawn without the special treatment to designated bidders, and whether entry would have become a problem there as it did in Europe.

Entry was also addressed in the UK auction, where the best licence of the five was reserved for an entrant. It appears from this straight-forward manner that both concepts of entry were in the minds of the auction designers. The UK telecom market was unusual in the sense that it was believed that there is room for a fifth 3G operator in addition to the four 2G incumbents. In such a situation the two concepts of entry are naturally tied together. The mere offer of a fifth licence ensures that at least one non-incumbent will win a licence, while the restriction of the bidding on one specific licence to entrants only, supplies them with some protection from the fierce competition incumbents are likely (and did) to engage in, and thus makes participating in the auction seem like an attractive business opportunity for newcomers. There was, however, a stage in the planning of the UK auction when it was thought that only four licences would be sold, in which case the question of entry would have had to be addressed in a different manner (indeed there were some alternative de-

signs considered for their entry-promotion properties, see Binmore and Klemperer, 2002 for a full account of the UK auction design).

The results of the UK auction suggest that its design was probably promotional to entry as the number of entrants in this auction was more than double the number of incumbents, an achievement in terms of entry that was not repeated in any of the European auctions that followed.

1.2.5 Creating an ‘Entry-Friendly’ Telecom Market

It is important to remember, when discussing possible ways a telecom auction could be made more attractive to newcomers, that the auction is merely the selling mechanism of a product that could be described as the right to operate in the telecom market. The environment and regulations that exist in the telecom market are therefore features of the product itself, their design in a way that is ‘friendly’ to the newcomers could therefore be a much more powerful tool in attracting entry (of both types). Steps in this direction could include simplifying the process of building up infrastructure and insuring roaming rights, however these are decisions usually made by policy makers and carried out by regulators, and not economists, and thus lie outside the scope of this study.

We will now turn back to the promotion of entry by means of the design of the telecom auction, and in particular to the method that will be the main concern of this study in what follows.

1.3 The Use of Royalties and Royalty-Bidding

Non-incumbents in the telecom market are viewed as weaker bidders. The source of their weakness is not, however, simply that the same licences are worth less to them than to their incumbent counterparts, as telecom licences draw their values, at least in the most part, from the opportunities that exist in the telecom market, and thus are common to all bidders. As was already mentioned above, a more likely source for the weakness of newcomers is the cost of building the telecom business. This cost would in most cases be smaller for the incumbents in the market who already hold some sites for radio masts, have an established consumer base and a brand name in the industry. However, these costs are still very significant even for incumbents, especially when a new market is targeted (as in the 3G case), and it is rather difficult to estimate the exact magnitude of the incumbents' advantage in this respect. How much discount should the auctioneer offer to entrants in order to offset this advantage? How can she avoid over-compensating the entrants and distorting the result of the auction by reversing the situation and making the entrants artificially better off, and thus making the auction vulnerable to speculators? Any choice of discount made by the auctioneer before the auction would seem arbitrary. It is especially problematic in the 3G auctions where a single auction is run and there is no real chance to 'correct' the rules of the auction, (as was in fact done in the US with respect to the treatment to designated bidders).

The search for a different method to lessen the advantage that incumbents hold over entrants in telecom auctions suggests the possible use of royalties. Royalties have been widely used as an additional payment to auctioneers in auctions of goods whose values depend heavily on the unknown conditions in a relevant market. (In the next chapter the study and use of such auctions is included in the review of the auction literature.)

The use of a pre-set royalty rate to be paid to the auctioneer, in addition to the cash payment paid at the end of the auction, serves as a means of reducing the affect of the differences in valuations before the auction, on the bidding process. It therefore acts in favour of all weak bidders to some extent; the actual competition however, remains in cash terms and the incumbents' advantage is not fully offset, but rather it is reduced.

We now want to take a more thorough look at a different possible use of royalties in auctions – the use of royalty bidding.

1.3.1 Basic Underlying Assumptions

As was already mentioned above, at the basis of the need to build into the auction a special treatment for newcomers, in order to encourage them to enter, is the assumption that their starting point in the bidding competition is weaker, i.e. that they would have to bear a higher cost than incumbents if they win the auction. The auction could therefore seem like a rather poor investment to non-incumbents, as they have to pay

the high entry cost but have only a small chance of actually winning the auction. An auction design that is favourable to entry would optimally make the auction more attractive to entrants, by increasing their chances of winning, but would not reach the point where entrants can manipulate the rules of the auction to their advantage. It would therefore be useful to have a more specific account as to the sources of the potential entrants' disadvantage – the source of these higher costs that entrants face - so that a more precise tool could be used for treating potential entrants.

We start by noting that, particularly when a new market is to be created, costs are likely to be quite significant even for incumbents. It is therefore often the case that both incumbents and entrants have to turn to the credit market for financing their investments, if they decide to bid in the auction. The incumbents' advantage as established firms already operating in a related market, is therefore likely to take the form of a lower interest rate on their borrowings. This higher price entrants are assumed to have to pay in the credit market for their loans, would make them more sensitive to negative results in the telecom market.

It is this 'increased sensitivity to bad outcomes' of entrants that we want to focus on as the more likely source of their disadvantageous position, rather than the wide assumption that all entrants face a higher cost (and thus a lower valuation) than incumbents. In other words, *we assume that entrants are more risk averse than incumbents*. As to their fixed costs, these are likely to be higher but could in some cases be lower than the incumbents'.

1.3.2 How Would a Risk-Averse Bidder Behave?

In order to isolate the affect of bidders' attitudes towards risk on their view of the auction as a worthy investment, we want to assume for the moment that no additional costs, other than the costs of entry, are to be born after the auction by any of the bidders.

Consider the following example: A single licence is on auction; two potential bidders exist, one is risk-neutral and the other is risk-averse. The licence will be worth, either π_L , if market conditions after the auction turn out to be bad, which happens with probability p_L , or π_H , if conditions are good, with probability $(1 - p_L)$. Both bidders know this in advance. Now we want to compare different auction designs in terms of their attractiveness to the risk-averse bidder. First we look at the ordinary (or cash) second-price, sealed bid auction. It is well known that bidders have a dominant strategy in the second-price auction, which is to bid their true valuations.

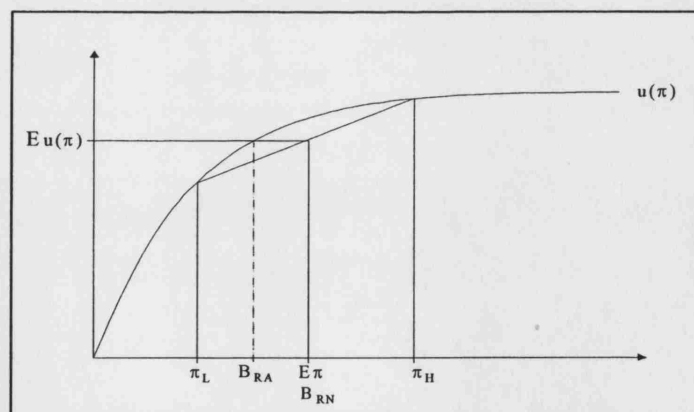


Figure 1: The 'cash' second-price auction.

The risk-neutral bidder will therefore bid the expected value of π (the amount B_{RN} in the Figure 1), while the risk-averse bidder will bid his certainty equivalent to that (the amount B_{RA} in the figure), which is lower for every level of risk aversion. If entering this auction is costly it is clear that the risk-averse potential bidder will choose not to enter, as he has no chance of winning.

Now consider the case where the winner of the auction would have to pay some percentage in royalty payments as the value of the licence is realised.

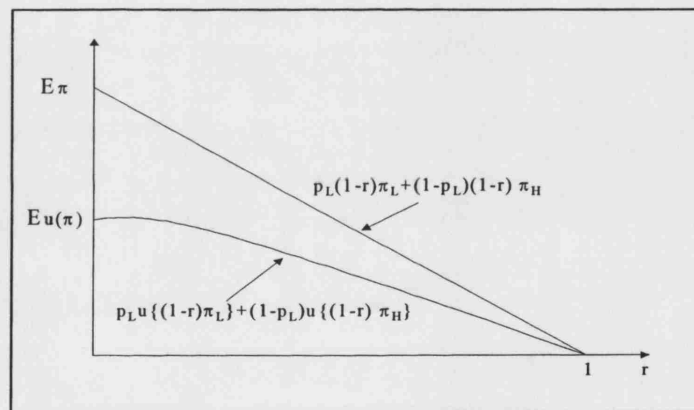


Figure 2: The 'pure' second-price royalty auction.

Figure 2 shows the expected utility of both bidders for different possible levels of royalty. We continue to assume for the moment, that a cash second-price auction is run to select a winner, who will then have to pay a pre-set rate of royalty payments, in addition to the cash payment of the second-highest bid. Bidders' bidding behaviour in this auction is quite similar to that in the previous example - the risk-neutral bidder

will bid her expected utility (which for her is equivalent to the expected payoff), while the risk-averse bidder will bid the certainty equivalent of his expected utility. As is clear from Figure 2 the difference between the expectations of the two bidders' utilities is decreasing with the royalty rate. That is, as the share of the payment that is determined after uncertainty is resolved increases, the risk-averse bidder is better off relative to the risk-neutral one, up to the point where all disadvantages disappear with $r = 1$.

The reason for this is that the good on sale here is effectively a lottery ticket; bidders' attitudes towards risk can therefore be thought of as their valuations for participating in this lottery, with the risk-averse bidder having a lower valuation than the risk-neutral bidder. By allowing some of the payment to be contingent on the outcome of the lottery the auctioneer is bearing some of the risk, an act that naturally is more appealing to the risk-averse bidder.

With the pre-set royalty rate, however, the auction designer still faces the problem of how to determine the right rate of royalties, so that it is high enough for the risk-averse bidders to choose to enter, but not too high, as royalty rates are known to distort the market when they are applied to bidders' revenues, (rather than to their profits, which are typically assumed to be effectively unobservable in the framework of a telecom market).

1.3.3 The Use of Royalty-Bidding

An auction that is run in terms of royalty rates, is one where the bidders' bids correspond to shares in their revenues (0% - 100%) that they are willing to pay the auctioneer after uncertainty in the market is resolved, whatever the outcome may turn out to be, rather than bidding in terms of cash sums to be paid after the auction but before the value of the licence is revealed. This way, the effective royalty rate is endogenised and set by the strength of the competition between bidders.

However, such a 'pure' royalty auction creates a different problem for the auctioneer. Assuming the framework remains that of a second-price auction, where bidders bid their true valuations, in the pure royalty case this would mean that both bidders have an incentive to raise their bids up to the point where they agree to pay the auctioneer 100% of their revenues (recall - we still assume no costs). A royalty rate of 100% (or close to it) is much higher than desirable for several reasons. First and most important is the fact that royalties distort the market. Unlike the simple example above where the value of the licence is given, in the fuller more realistic model the value of the licence is dependent on the winner's actions in the market. Since profits in the telecom market are usually assumed to be unobservable by the government, any royalty rate would have to be applied to the winner's revenue, and thus would cause a distortion to his or her optimal production decisions in the market. The higher the royalty rate applied the bigger the distortion. A second point to consider is the auctioneer's revenue from the pure royalty auction. As is the case with

other proportional taxes, the revenue from royalties will probably start decreasing as the rate of royalties becomes too high.

We can now state the problem in the core of this study, the one that will be addressed more formally in the third and fourth chapters below. An auctioneer whose priority is to promote the entry of risk-averse bidders into the auction will want to determine the lowest possible royalty rate that will make them choose entry, and then apply this rate to the winner of the auction³

1.3.4 The Second-Price Royalty Auction with a Fixed-Fee Element

As was noted above the pure royalty auction is likely to generate a very high royalty rate, while the constant royalty rate, pre-set before the auction, is likely to be arbitrary, and could be too high or too low for the task of promoting entry of risk-averse bidders into the auction. We therefore examine a ‘second-price royalty auction with a fixed-fee element’.

Such an auction is the object of the analysis carried out below, and it will be fully defined and presented within the framework of the model. At this stage we want to give only a brief description of its main features. We therefore turn back to the real world to consider the experience of a 3G telecom auction run in terms of royalty-rates - the Hong Kong 3G auction.⁴

³ We argue later that such a system of priorities may often be realistic in the telecom market environment.

⁴ Although the Hong Kong auction is by no means *equivalent* to our ‘second-price royalty auction with a fixed-fee element’. The similarities and differences will be elaborated on in chapter 4.

A second-price royalty auction with a fixed-fee element is an auction run in royalty terms, but in which every possible royalty bid corresponds also to a cash amount, that the bidder is committing to pay the auctioneer in addition to the royalty payments. The schedule of cash amounts is designed by the auctioneer in advance of the auction, and is a tool in her hands to influence bids, where, roughly speaking, a higher cash amount means a lower royalty bid.

Before we turn to a brief examination of the only (to the best of our knowledge) real world use of royalty bidding in telecom auctions, it is important to highlight a certain aspect of the auction we develop and analyse in this work. This is the aspect of its efficiency, in its usual sense of the winner of the auction always being the bidder with the highest valuation for the object on sale. A second-price royalty auction with a fixed-fee element could result in an inefficient allocation of the licence in that sense. This is a natural ‘by product’ of the attempt to promote the participation of risk-averse bidders whose valuations of the good we referred to as a lottery above is naturally lower.

Making the auction more attractive to the weaker bidders by increasing the chances of their winning it, is effectively giving those weaker bidders a chance they otherwise would not have, this must create the possibility of inefficiency. In other words, the promotion of entry must involve a certain sacrifice of efficiency. If no inefficiency is to be introduced, then the ordinary (cash) second-price auction is as good as we can reasonably hope for.

Moreover, the basic environment of a telecom auction is one where asymmetries among bidders' valuations are a natural assumption, and it is well known that, in the presence of asymmetries, some of the most commonly used auction formats (and in particular the first-price auction and its derivatives) may result in inefficient allocations (in the next chapter a fuller account of auction theory in the presence of bidders' asymmetries is presented).

In what follows, the question of efficiency of the royalty auction does not take central stage in the theoretical analysis of chapters 3 and 4, as some degree of inefficiency is unavoidable in a setting where entry is to be promoted.

The only place where an auction with some similarities to the second-price royalty auction with a fixed-fee element was actually used is Hong Kong, where in July 2001 a spokesman for the Information and Broadcasting Bureau announced that: "Recognising the recent downturn in the telecommunications market, we have introduced a royalty-based payment scheme that is intended to minimise the financial burden on operators. The royalty scheme is underpinned by a schedule of minimum payments, which minimise the government's credit risk but allows it to share the upside of the 3G business". The spokesman added that: "In setting the reserve price, we aim to encourage entry to the auction, but are also mindful to set a reasonable minimum price for a scarce public resource like spectrum". (Press Release Issued by Information Technology and Broadcasting Bureau (ITBB) on July 18, 2001)

The HK auction did have some similarities to that studied here but, it was by no means equivalent, and had some very important differences too. A fuller description and some analysis of the HK auction is presented in chapter 4 below. Only its main features and results are given here.

Four licences for operating 3G technology were put on auction in HK. Bidders were to bid in terms of royalty rates, where the winners would be the 4 bidders who bid the 4 highest rates. The resulting effective royalty rate applied to all winners and was the 5th highest bid. This means that the HK auction was in fact a second-price (or highest-loser) royalty auction⁵. The “schedule of minimum payments” mentioned by the spokesman corresponds to our ‘fixed-fee element’ and is a schedule of cash payments, but as opposed to the auction analysed here winners in the HK auction had to pay either the royalty payment on their revenues (the actual term used in the HK memorandum was ‘network turnover’), or a cash payment, whichever ever turns out to be the highest, but not both. The HK auction took place in September 2001. Although the HK case was unique in having more incumbents than 3G licences - at the time of the auction there were 6 second generation operators in the market (reduced from the original 8) - only 4 leading incumbents applied for participation in the 3G auction. The 4 licences were therefore sold to the 4 incumbents for their reservation royalty rate, which was 5%.

⁵ The original plan was to promote entry by using a lowest-winner format. That was later overturned.

Chapter 2

Theoretical Literature Survey

In the previous chapter we provided the practical basis for this study. We looked at the telecom market environment, and at the role telecom auctions take in it. Real-world experience revealed that in telecom auctions the issue of bidder participation is at the heart of the perceived success of the auction, and present a real challenge for designers of the auction rules. These considerations motivated the idea of a ‘second-price royalty auction with a fixed-fee element’ as an attempt to capture as many of the properties of the real telecom environment as possible, albeit in a simplified manner, and provide a framework in which the question of bidder participation could be addressed and studied.

We now want to turn to the theory. The fact that a second-price royalty auction with a fixed-fee element was constructed to enable us to study the specific question of entry within the special environment of a telecom auction, means that theoretical studies where all its elements are combined are nonexistent. On the other hand, the literature that is related in a more partial way, where a certain aspect of the relevant environment is modelled and studied, is vast. We therefore divide the literature into four branches, each of which studies some feature at the basis of our study. These four branches are:

- Auctions with entry.

- Auctions with risk-averse bidders.
- Auctions with asymmetric bidders.
- Auctions where the winners' payment is not a function of bids alone.

Each of these literatures will be surveyed separately in an attempt to present its main results, and draw relevant links with the question studied here.

2.1 Auctions with Entry

Entry of bidders into auctions - how it can be influenced and the costs of such actions - are the main focus of this study. Our experience of telecom auctions has shown us that, in this framework, entry could become problematic, and in most cases would need to be promoted through the auction design in order to ensure that the auction achieve its goals. This observation stands in contrast to the usual assumption made in auction literature that the number of bidders is fixed and independent of the auction format. Attempts to relax the fixed number of bidders assumption have been made in several frameworks. However, most studies which take this approach are concerned with the effects of a variable number of bidders on the auctioneer's revenue. The level of entry is regarded as an objective only in so far as it increases the auctioneer's revenue. In the telecom auctions environment this is not always the case. On the contrary, entry, or entry-related auction performance, may actually take priority over the raising of revenue in the eyes of some telecom auctioneers. Nevertheless we think

it useful to state the main results of the literature on auctions with entry, as some of the forces present in these broad models will be relevant in our analysis of the special telecom case.

Among the early studies to deal with entry is Milgrom & Weber (1982) who included entry fees in their general model of affiliated bidders' valuations. They study the effect that an entry fee, which they assume is set by and paid to the auctioneer, has on the auctioneer's revenue. As opposed to the situation studied in this work, the entry fee in Milgrom & Weber is an additional tool in the auctioneer's hands, and she uses it (as she uses a reservation price) to extract revenue from risk-neutral bidders who know their types (or signals) before hand. They do note however, that in some cases, imposing an entry fee (as opposed to a reservation price) may cause the model not to have a monotone equilibrium in the entry stage, where all bidders with signals above some threshold choose to enter.

McAfee & McMillan (1987a) analyse an independent private-valuations model where risk neutral bidders have to decide whether or not to pay an entry fee and participate in an auction, before they learn their types. In this framework bidders' expected profits negatively depend on the number of participants, as the chances of a bidder winning the auction are greater the fewer the bidders who participate in it. Bidders will therefore choose to enter as long as their expected profit exceeds the entry fee. McAfee & McMillan define the optimal number of bidders in an auction as the number that maximises the auctioneer's revenue. They find that a first-price

sealed bid auction whose reservation price equals to the auctioneer's own valuation (as opposed to a reservation price which exceeds the auctioneer's valuation which is typically optimal in a setting with no entry), combined with a given level of entry fee will induce the optimal number of bidders to enter.

Engelbrecht-Wiggans (1993) studies a model in which the auctioneer sets a bidder-specific entry fee, which could be negative and interpreted as a subsidy (although bidders also incur a cost of bidding which is always greater than such a subsidy). Risk-neutral potential bidders then have to decide whether or not to participate in the auction before they learn their valuations for the object being sold. The optimal number of bidders in the auction is defined as the number for which expected gains from trade are maximised. Engelbrecht-Wiggans finds that the auctioneer can ensure that the optimal number of bidders will enter, while her revenue is maximised, if the auctioneer sets a combination of a reservation price which equals her own valuation for the object together with an entry fee that is designed to push the marginal bidder's expected profits exactly to zero. The author is concerned mainly with the trade-off that exists between the two different tools available to the auctioneer, namely a reservation price and an entry fee. He finds that when both tools are available for the auctioneer she might prefer to use an entry fee/subsidy rather than an optimally set reservation price (which is typically higher than the auctioneer's own valuation), as long as the set of bidders that choose to enter does not change.

Levin & Smith (1994) construct a model where risk-neutral symmetric potential bidders choose whether or not to pay a constant cost of entry, (which is not set by or paid to the auctioneer), and bid in the auction. If they choose to enter they learn the total number of bidders that also chose to participate, the exact rules of the auction, and their own estimate for the item's value, before they bid. Among the 'rules of the auction' the auctioneer may specify an additional entry (or admission) fee, and/or a reservation price. They construct a symmetric equilibrium in mixed strategies in which all potential bidders enter with the same positive probability. The equilibrium probability of entry is that which makes the bidders' expected payoff, given the auction rules and number of participants, equal zero. This requirement of their model means that the auctioneer's revenue is the only measure for social welfare. In the independent private-value model, the authors find that welfare is maximised with free entry (i.e. no additional means of restricting entry – reservation price or entry fee – are imposed). In the common-value model, however, welfare is maximised when the auctioneer sets an additional entry fee and restricts entry. In general they find that when values are affiliated a second-price auction always generates less entry than a first-price auction. They also extend the revenue-equivalence theorem to the case of a variable number of bidders.

Kaplan and Sela (2003) study an environment where bidders' valuations, (which are not necessarily symmetrically distributed), are common knowledge, while their costs of entering the auction are private information. Bidders have to decide whether

or not to enter a second-price auction after they learn their cost. They find that the model has a cutoff type equilibrium (which is not necessarily unique). In equilibrium only bidders with costs below the cutoff level enter the auction. In the asymmetric case the (second-price) auction is not necessarily efficient, moreover, the seller's revenue might be maximised in equilibrium if only bidders with *high* costs were to enter. In any case the authors show that the seller is always better off if participation in the auction, of at least one type of bidders, is reduced.

2.2 Auctions with Risk-Averse Bidders

In the previous chapter, we discussed the importance of the assumption regarding entrants' risk-aversion, as a possible source of their weaker position at the beginning of the auction. More specifically we considered an environment in which some bidders are more risk-averse than others. This heterogeneity in bidders preferences is crucial in our setting since it is the known difference in the bidders' position in the auction that drives the problematic issue of bidder participation in telecom auctions. This framework, however, stands in contrast to what is usually assumed in the theoretic literature with regard to bidders' attitudes towards risk. The majority of auction studies deal with risk-neutral bidders, but even when the risk-neutrality assumption is relaxed, homogeneity is usually preserved, and bidders are all assumed to possess the same degree of risk aversion.

The extension of the basic auction model to include bidders' risk aversion complicates matters considerably, since an auction, even at its simplest form, creates a risky environment for bidders, and thus the assumptions made regarding their attitudes towards risk may affect their behaviour directly. The most basic auction situation is one where a bidder's bid determines only his or her probability of winning, but not the winning payoff (as in a second-price or a Vickrey auction). This is a risky environment, but as the bidder's task here is very straightforward, optimal behaviour is accordingly simple. It is when bidders' bids determines both their probability of winning and their payment if they win, that things become complicated, as a trade-off then arises between increasing the probability of winning and the rent gained upon winning. For this reason the literature on auctions with risk-averse bidders deals mainly with first-price auctions, where such a trade-off does exist, while in the framework of the second-price auction, where the winner's payment is independent of his or her bid, bidders' attitudes towards risk should not alter their bidding behaviour.

Although the model developed here rests essentially within the second-price framework, the main results on auctions with risk-averse bidders are fundamental in the auction literature, and are illuminating in the understanding of the risk-averse bidder's task, we shall therefore present them here, albeit in brief.

Holt (1980) analyses a model of competition for procurement contracts. He considers two alternative procedures, one is an open ('competitive') auction, and the

other is a sealed-bid first-price ('discriminatory') auction. The symmetric bidding equilibrium with possibly risk-averse bidders is derived, and a comparison is made between the expected procurement costs under each alternative procedure. Within this framework, Holt demonstrates the importance of the difference in the structure of information between the two auction formats when bidders are risk-averse. Under the sealed-bid auction, unlike the open-bid procedure, bidders' do not acquire any additional information in the bidding process, and thus they behave as if they are facing a riskier environment when participating in a first-price auction. This causes the risk-averse bidders to reduce their bids in the first-price auction, and the expected procurement costs drop by the amount of the expected risk premium that the winning bidder will be willing to pay in order to avoid the risk.

Riley & Samuelson (1981) derive a similar result in the setting of an auction of a single indivisible good with symmetric, possibly risk-averse bidders. They also show that, while the optimal bidding strategy of bidders in the second-price auction remains bidding their true valuation for the good regardless of their attitude towards risk, risk-averse bidders in a first-price auction bid uniformly higher than their risk-neutral counterparts. This breaks the seller's revenue-equivalence, and makes the first-price auction take the lead in terms of revenues to the seller, when bidders are risk-averse. The authors show also that in the revenue optimising auction with risk-averse bidders, the optimal reservation price is decreasing with the degree of bidder risk-aversion. That is, if the seller is viewed as an additional bidder whose bid is

the reservation price, in the first-price auction not only the buyers' bids get closer to their true valuations as they get more risk-averse, but also the seller's, who sets a reservation price which is closer to his or her own valuation, even though the seller is assumed to be risk neutral.

Maskin & Riley (1981) elaborate on the design of the optimal auction - the auction that maximises the seller's expected revenue -- in the case of risk-averse bidders. They show that, in the independent private-valuation framework, the seller can exploit the bidders' risk-aversion to increase his or her gains further by requiring payment from losers as well as from the winner. That is, the seller should charge an entry fee from potential buyers who wish to participate in the bidding. The intuition provided for this line of thinking is that introducing an entry fee makes the auction environment more risky; as a result, bidders will reduce their bids regardless of their attitudes towards risk, but the risk-averse bidders would do so by less, as they are willing to pay some positive risk premium to insure them against losing. It would therefore be possible for the seller to set the entry fee at such a level that the gain from all participants outweighs the loss from the drop in the winner's bid. A similar result is also provided for the common-value framework, (with some restrictions placed on the coefficient of absolute risk aversion).

Maskin & Riley (1984) complete this earlier analysis of optimal auctions with risk-averse bidders. The expected-revenue maximising auction, when bidders are risk-averse is shown to be designed in a way that balances the tendency of bidders

to increase their bids (in the first-price auction) the more risk-averse they are, with their willingness to pay in advance for a reduction in the degree of risk they face. This leads to an auction where low bids are riskier to make than high ones, and so high valuation bidders are encouraged to keep their bids high. Only the bid that corresponds to the highest possible valuation is not risky at all, i.e. the bidder with the highest possible value is offered complete insurance.

Moore (1984) characterises the optimal auction with risk-averse bidders, under a somewhat different set of assumptions, and with a single buyer. He finds that the bidder in the optimal auction is always better-off winning than losing. That is, there is some uncertainty left unresolved even though the winner is offered the opportunity to buy some insurance against it. He also shows that the winner's expected payment and therefore the seller's expected revenue are non-decreasing functions of the bidder's valuation for the good on sale. If the single risk-averse bidder has a constant coefficient of absolute risk aversion then his or her payment to the seller, in the optimal auction is the same whether he or she wins or loses.

Matthews (1987) studies several alternative sets of auction rules from the risk-averse bidders' point of view. Among his results is that the change in bidders' coefficient of absolute risk aversion (decreasing, constant, or increasing) is central in determining whether they prefer to participate in a first-price or a second-price auction, (in the sense of which one offers them a higher expected payment if they win). In particular, bidders prefer the second-price to the first-price auction if they have

decreasing absolute risk aversion; they are indifferent between the two if their coefficient of absolute risk aversion is constant; and they prefer the first-price auction if it is increasing.

And finally, Fidich, Gavious, and Sela (2004) compare a first-price auction and an all-pay auction in terms of the bidding strategies used by weakly risk averse bidders. They concentrate their attention on weakly risk averse bidders as this allows the use of perturbation analysis in order to approximate a closed form for the bidding functions, something that is very difficult to achieve in these auctions in the presence of bidders' risk aversion. The authors find that in the first-price auction risk averse bidders bid more aggressively than risk neutral ones. In the all-pay auction on the other hand, risk averse bidders with low valuations bid less aggressively than their risk neutral counterparts, while bidders with high valuations bid more aggressively, even more so than they would in the first-price auction.

2.3 Auctions with Asymmetric Bidders

One of the important features of a telecom environment as modelled here is the existence of asymmetries among the bidders. More specifically, we claim that telecom auctions could not be fully characterised and understood without taking account of the fact that, in such an environment, some bidders have an advantage with respect to others at the outset of the auction. This advantage is expressed in two different aspects of the model; the assumption that some bidders (the newcomers or entrants)

are more risk-averse than others, and the assumption that some bidders (again the entrants) have costs that are likely (though not certainly) to be higher than their incumbent opponents.

The first of these assumptions, asymmetries in attitudes towards risk, is a crucial assumption in our model, the second, asymmetries in distributions of valuations, is not a central assumption and in fact most of the qualitative results would remain even if it were relaxed. However, differences in attitudes towards risk lead to differences in valuations of risky monetary payoffs. Therefore, an analysis of an environment where payoffs are certain and bidders exhibit asymmetries in the distributions of valuations, could in fact shed some light on the nature of asymmetry that is in the centre of our study.

The literature on auctions with asymmetric bidders is concerned primarily with asymmetric distributions of valuations, and is sometimes technical in nature. In the remainder of this section we will therefore present the main results of this branch of the literature, without attempting a complete analysis of the statistical properties of bidder asymmetries.

Maskin & Riley (2000) analyse several possible structures of bidders' valuations, all of which result in asymmetries among bidders. They then derive the equilibrium bidding functions, and compare the performance (in terms of expected revenue for the seller) of the first-price and second-price auctions. All of the structures that are considered, however, are such that bidders could be either of a strong type or

of a weak type, in the sense that the distribution of the strong bidder's valuations conditionally stochastically dominates that of the weak bidder. The authors then prove, among other results, that in the first-price auction a weak bidder who is faced by a strong bidder will bid more aggressively than he or she would when faced by another weak bidder, in the sense that the bid in the former case stochastically dominates the bid in the latter case. Moreover they find that in a first-price auction a strong bidder will bid less aggressively when face with a weak bidder, than he or she would when faced with another strong bidder. In contrast to these results, it is well known that in a second-price auction it remains a dominant strategy for all bidders to bid their true valuations even in the presence of asymmetries. These two results could therefore have an important affect on entry issues when bidders are asymmetric, as they imply that weak bidders who know they are faced by strong bidders will have a greater incentive to participate in a first-price rather than a second-price auction as their chances of winning it are greater. From the seller's point of view Maskin & Riley find that when the strong bidder's distribution of valuations is either a shift to the right or a stretch to the right of that of the weak bidder, then the first-price auction generates higher expected revenue than the second price auction. The opposite however is true when the weak bidder's valuation is similar to that of the strong bidder only with a fraction of the density shifted to the lower end point.

Cantillon (2000) examines the effect that the presence of asymmetries among bidders has on the seller's and bidders' expected payoff from a first-price and a

second-price auction. For the purpose of isolating this effect she constructs a benchmark auction, where bidders are symmetric, and which is equivalent to each of the asymmetric auctions in terms of maximum surplus attainable in them. Cantillon's results show that in the framework of the second-price auction the auctioneer's expected revenue is less when bidders are asymmetrically distributed, in comparison to that of the equivalent (in the sense defined in Cantillon 2000) symmetric second-price auction. The first-price auction environment is much more complex and a similar result could be proven only under some restrictions on the structure of the asymmetries, she does however conjecture that the more general result, - that the auctioneer is worse off in the presence of asymmetries in the first-price auction - holds as well. Bidders in the second-price auction are shown to gain, at least in the aggregate, from being asymmetrically distributed. In the framework of a first-price auction a definite result of this nature is harder to derive since its outcome is possibly inefficient in the presence of asymmetries, however it is shown that both bidders and the auctioneer could be made worse off by asymmetries in the first-price auction.

Kaplan & Zamir (2002) argue that the important type of bidder asymmetry is not so much differences in bidders' distributions of valuations, but rather differences in bidders' beliefs about these distributions. In this framework, a symmetric auction is one where all bidders hold the same beliefs about the distribution of their opponents' valuations, while in the asymmetric counterpart different bidders hold different such beliefs. They address the question of whether such asymmetries among

bidders affect the seller's expected revenue from the first-price auction, and in what way. The authors also examine whether it changes the ranking of the first-price and second-price auctions in terms of the expected revenue they generate for the auctioneer. The framework in which these questions are addressed is one of a game of incomplete information where the information about bidders' distributions of valuations, which in fact could either be symmetrically or asymmetrically distributed, is either revealed to them (in the asymmetric case) or kept from them (in the symmetric case). The authors find that the two questions are connected in the sense that asymmetries in bidders' beliefs could increase (decrease) the seller's expected revenue in the first-price auction, depending on whether the first-price (or second-price) auction performs better when the asymmetries are present in the distributions of valuations, but with no incomplete information.

Finally, Marshall et al. (1994) provide a numerical analysis of equilibrium in the first-price auction with asymmetric bidders, as a means of avoiding the highly intractable nature of the analytical approach. Specifically, they look at asymmetries in the distribution of valuations that result from a symmetric environment in which some of the bidders may collude and form a coalition that bids together. An interesting result of their analysis is that the equilibrium bidding functions of bidders in this environment are equivalent to the bidding functions that would be used in an environment where there are no distributional asymmetries, but instead bidders possess different degrees of risk aversion.

2.4 Auctions where the Winners' Payment is not a Function of Bids Alone

In this study, our main objective is to investigate the effects of the use of royalties in telecom auctions on the behaviour of both potential bidders, and the auctioneer. Introducing royalties into auctions payments changes one of the key features of the basic auction model studied in the literature – that winners' payments are derived from their bids alone. Moreover, royalties could be a meaningful tool at the auctioneer's disposal only when at least some component of the good's value can be determined only after the auction is over.

Auctions with royalties have been used in practice in the past; for example, in auctions of oil rights and publishing rights. The literature on such auctions is quite broad, since it has both theoretical and empirical aspects, and could in fact include the vast literature on optimal contracts in licensing, where the use of royalties is examined but the auction stage and its design, if studied at all, plays an insignificant role. However, the question of whether the availability of royalty payments affects bidders' behaviour, and in particular their decisions regarding participation in the auction, has not been addressed in the literature, to the best of our knowledge.

We exclude from this section most of the literature on optimal contracts, as our main concern is with the design of an auction using royalty payments. However, some papers in this section will be presented in somewhat more length since,

although most assume bidders are symmetric and risk-neutral, nonetheless some provide an important insight into key issues for this study.

Leland (1978) considers the process of the leasing of natural resources via an auction. Firms are assumed to be risk-averse and the winner's actions after the auction affect (together with the realised state of nature) the value of the tract. Players are, however, all symmetric – no private information is introduced. The seller receives two forms of payment, the winner's bid and the appropriate sum of royalty payments, determined according to a schedule specified in the lease, and which may depend on the tract's value, i.e. the lease specifies a schedule of royalty rates, as a function of the tract's realized value. Leland then turns to characterising the revenue maximising payment schedule under different assumptions regarding the firms' actions. Bidders are assumed to bid their true valuations, i.e. the amount that equates the expected utility from the difference between their net returns from the risky investment, and their bid to the utility they derive from zero. This is assumed although the cash component of the winner's payment is his or her own bid, i.e. the auction is of the first-price structure.

Leland assumes first that the firm's actions after the auction are exogenous (independent of the payment schedule). The shape of the optimal schedule is found in this case to vary inversely with the ratio of the auctioneer's to the bidder's absolute risk aversion. In particular, if the auctioneer is assumed risk-neutral then the optimal schedule is linear with a 100% royalty rate, i.e. the winning firm is paid a fixed

amount for extracting the natural resource, and all profits from the tract go to the auctioneer. More generally Leland finds that the optimal schedule will be linear, i.e. will include a fixed fee together with a royalty element, only if the seller and the firm feature the same slope to their risk tolerance (the inverse of absolute risk aversion) curve at the payment point. If both the seller and the firm have logarithmic utilities, but the seller is less risk averse, then the optimal schedule is convex. An increase in the expected value of a tract should cause an increase in the slope of the optimal schedule, i.e. the royalty rate for every value of the tract is increased. A similar affect is associated with the firm becoming more knowledgeable about the value of the tract than the seller - the optimal schedule becomes steeper.

Assuming that the firm's actions are no longer exogenous, but are determined optimally, Leland provides some results on when the optimal schedule can achieve the Pareto optimal production level. When the royalty system is based on the firm's revenues, Pareto optimal production can never be achieved, as a Pareto optimal schedule is possible only if it does not affect the firm's actions after the auction. A Pareto optimal schedule is based on the true realized value of the tract, and can be linear, but a convex (concave) schedule will lead the firm to under (over) produce.

While Leland (1978) presents some very interesting and quite broad results regarding the properties of the optimal payment schedule, the setting he considers makes some of the questions studied in this thesis impossible to address. First and most important, Leland does not actually model the bidding competition, and since

there is no private information, this is equivalent to assuming only one bidder exists. Naturally the question of whether or not to enter the auction, which is the focus of this study, cannot arise. Secondly, the payment schedule, and specifically the royalty rate, is specified in the lease and depends on the value of the tract, on which the bidder has no influence at the time of the auction. The auction (which is not actually modeled) is a first-price auction (although bidders are assumed to bid truthfully), and the optimality of the payment schedule is measured only by its effect on the seller's revenues.

The literature on optimal contracting in principal-agent problems often deals with the construction of an optimal payment schedule. Busquet, Cremer, Ivaldi & Wolkowicz (1998) characterise the optimal contract between a single, risk-neutral seller of a patent to the single, risk-averse, buyer. No auction takes place in this setting. Two of their results are as follows. A fixed-fee contract is never optimal with a risk-averse buyer, i.e. some level of risk sharing through royalties is optimal. Under demand uncertainty the optimal contract involves an ad valorem royalty system, as opposed to a per-unit royalty rate.

A different approach is taken in Riley (1988), who considers an interdependent valuations auction model, where bidders are symmetric and risk neutral. He then makes alternative assumptions about the payment rule in the auction, (in each case however, the winner's payment is not dependent on bids alone), and examines the effect that this has on the auctioneer's revenue. One of the settings Riley considers,

and the one most closely related to our model, is that of the pure royalties auction, where bidders bid in terms of royalties and the highest bidder wins and pays his bid times the value of the item, which is derived from the realisation of a random variable. He finds (proposition 6) that the expected payment of the winning bidder in the pure royalties auction is always greater than that in the equivalent first-price cash auction. It is perhaps interesting to note at this point that some results in this thesis are in apparent contradiction to Riley (1988). There are three main differences between the model analysed here and that of Riley (1988). Firstly, Riley's setting does not explicitly model the market after the auction, so that the winner cannot affect the value of the item by his or her actions. Royalties in this setting are applied to output which is assumed to depend on some random variable which bidders can only estimate before the auction, and is revealed fully (after some cost is incurred) after the auction. It is clear that such a framework assumes away the distorting effect that royalties have on production decisions, which may in turn reduce output, and thus the auctioneer's revenue. At the concluding part of the paper, however, Riley does remark that in a model where the level of output is in fact chosen optimally by the winner after the auction, the seller's revenue maximising royalty rate is strictly positive.

This result relies heavily on what was termed by McAfee & McMillan (1986) the 'bidding-competition effect' of an increase in the royalty rate which an auction winner is due to pay. This effect makes bidders in a first-price auction with royalty

payments bid more aggressively as a result of an increase in the royalty rate. The intuition behind this bidding behaviour as McAfee & McMillan present it, is that an increase in the royalty rate reduces the asymmetries among bidders' signals, net of the ex post royalty payments (which are independent of the identity of the winner and his or her valuation), and thus it induces more aggressive bidding behaviour. This effect of an increase in the royalty rate is similar to the effect a reduction in the variance of the distribution of signals would have on the bidding. This leads us to the second important difference between the model developed here and that of Riley (1988). While we use the (fixed fee or cash) second-price auction as a benchmark, Riley's analysis is based on a first-price auction. In a first-price auction, the winner's ability to gain some rent is due to his private information, and a reduction in the significance of this private information causes a reduction of rents. This effect prevails up to the point where there is no private information at all, i.e. when all bidders share the same valuation for the item on sale, where each bidder would have the incentive to increase his bid until no rent gains are possible. In the second-price auction on the other hand the amount the winner pays is independent of his own valuation, and therefore of his private information. A reduction in the significance of private information will not induce the same effect on bidding behaviour in the second-price auction, in fact, following an increase in the royalty rate, it would still be in bidders' interests to bid their true valuations, net of the expected royalty payments.

And lastly, Riley's model assumes bidders are symmetric and risk-neutral. Intuitively, in this framework bidders are not likely to be able to gain from the introduction of royalty payments to the auction. We on the other hand, look specifically at a situation where royalties are used to improve the position of (some) of the bidders in the auction, naturally this could come at the expense of the auctioneer (at least in expected terms).

Hansen (1985) considers the independent private-values framework, with risk neutral bidders. No post-auction decision making is modeled, and so bidders information about the items value is unchanged as a result of the auction. Two alternatives to the common cash-bidding system are studied: Stock bidding for corporate control, where bidders bid in terms of the percentage of the merged entity that will be owned by the seller after trade occurs; and profit-share bidding for oil leases, where bidders bid in terms of the percentage of expected profits that they are willing to pay the seller. Profits, of course, would have to be assumed observable. Both auctions are of the open-auction format. In this setting, Hansen finds that both of these non-cash auctions generate higher expected revenue to the seller than the equivalent cash auction.

Reece (1979) considers three alternative bidding systems for offshore oil leases; bonus bidding, i.e. a cash first-price auction; profit-share bidding, and royalty bidding, where bidders bid in terms of royalty rates which will be applied to the true value of the tract, that is realised after the auction. All auctions are of the first-

price format, bidders are risk neutral, and no post-auction decisions are taken by the winner. In this setting, Reece finds that the seller's revenue is the highest under profit-sharing and lowest under bonus bidding. This relationship is enhanced when the value of the tract becomes riskier, or when bidders become fewer.

An empirical study that deals with some of these issues is that of Moody (1994), who provides an empirical analysis of the experimentation by the US government with alternative bidding systems for offshore oil tracts, conducted in the 1980s. One of the announced aims of these experiments, was to encourage small companies to participate in the bidding by reducing the up-front amount of cash required. Four alternative bidding systems were experimented with; a combination of cash bids and some sliding scale rate of royalties, varying with the level of production after the auction, between 16% and 65%; a combination of cash bids and some pre-fixed level of profit share, varying between 30%-50%; cash bids, combined with a pre-set 12.5% royalty rate on production; and cash bids combined with a pre-set 33% royalty rate on production. Bidders are assumed identical, that is they all have the same cost but they can decide whether to buy an estimate of the tract's value – their signal – which will then be known to them privately. Bidders are assumed risk neutral. The number of bidders participating in the auction is assumed to be that which equates the expected gain from winning the auction, calculated before bidders acquire a signal, with the cost of acquiring a signal. The entry decision is thus an indirect one, and it is assumed to be taken when all bidders are identical in all respects.

Moody's results show a positive relationship between the use of the sliding scale system and the number of bids received. No other bidding system showed any significant relationship to the number of bids. None of the alternatives was found to be associated with a lower (up-front) cash bids, which is taken to imply that the increase in the number of bids is not likely due to small companies in particular. The proportion of small companies (i.e. not in the top 20) was found to be the greatest in the traditional system (which was cash bids plus 16.67% royalty rate on production). The number of small companies was larger still if information on joint bidding (where several companies bid together) was included.

The study conducted in this work is primarily theoretical, but it stems from real-world experiences with telecom auctions, and thus aims to take account of as many 'real-world' features as possible. This was not done thoughtlessly, but in the hope that in a field such as auction theory, in which an opportunity is presented to observe the theory's predictions in real-world situations, and develop it in a more realistic direction, such an approach is interesting and meaningful. Naturally, this requires creating a model that unites different aspects of the literature that are typically studied separately. It is therefore difficult to build on the existing literature to provide precise guidelines for our task. Nevertheless, our study is one of auction design, a question that is addressed in the literature over and over again in many different settings and structures, and thus all of these provide the framework for our study even though no one fits perfectly.

Chapter 3

The Second-Price Royalty Auction with a Fixed-Fee Element the 'one incumbent - one entrant' case

We now turn to the main body of this study - the theoretical investigation of the use of royalty bidding in telecom auctions, in order to promote the participation of non-incumbents. A second-price royalty auction with a fixed-fee element, as introduced briefly in the first chapter, is the main object of our investigation. We present a model which aims at capturing the most important features of the telecom auction environment with a view to explaining the actions and reactions of all participants in the game. However, in trying to achieve this goal we have had to abandon the pursuit of a more generally applicable tool, as simplifying assumptions became necessary to make the issues at the centre of this analysis tractable. We shall try to point out these limiting assumptions and their possible consequences as we go along.

We now turn to the formal development and analysis of a second-price royalty auction with a fixed-fee element, first in the simplest setting that allows the relevant issues to arise; the one incumbent - one entrant case. In doing so we aim to answer the following questions:

- Is the introduction of royalties to telecom auctions promotional to entry?
- When is it advisable to use royalty bidding in the telecom auction environment?

3.1 Description of the Game

With a view to the Hong Kong experience, the central object of our analysis is a second-price royalty auction, where an additional cash component is introduced in order to balance between the distortion of the market, caused by the royalty payments, and the promotion of entry of risk-averse potential bidders. Before the auction is run in the second stage of the game, we allow non-incumbents to choose whether or not to enter and become bidders in the auction. This is the 'entry stage', where the success of the pursuit of entry can be measured. After the auction the winner operates in the telecom market, where his or her actions determine the level of production and royalty payments. Stages will be presented and analysed from third to first.

3.1.1 The Market

In the third stage the winner of the auction in the second stage operates the 3G technology, and serves as a monopolist provider of telecommunications services in the market, for one period. By the end of this period his or her revenue (R) is observed, and the appropriate share to be paid as royalties to the auctioneer is determined and transferred⁶.

The market for 3G services is a new market and therefore actual demand for these services is unknown until the beginning of the third (market) stage. The randomness of the demand is modeled using the random variable Δ , which is assumed

⁶ We assume, in accordance with the usual practice, that profits are unobservable.

to follow a Gamma distribution with density function: $f(\Delta) = \frac{1}{m!} \Delta^m e^{-\Delta}$, with mean $\mu = m + 1$ and variance $\sigma^2 = m + 1$. We assume that the inverse demand function, that gives price as a function of quantity produced, takes the form:

$$p = \left(\frac{\Delta}{q}\right)^{\frac{1}{2}} + \frac{p^0}{q}$$

It is already necessary at this stage to point out the simplifying assumptions made by assigning specific functional forms to variables. These are made for the purpose of simplifying the algebra, and are not crucial for the analysis. Special attention should be given to the term p^0 which is some (large) constant number that we introduce into the market demand in order to avoid the need to take account of possible bankruptcies that might otherwise occur when the market demand is revealed. A fuller analysis that includes the possibility of bankruptcies is briefly sketched and presented in appendix A.

The winner of the auction in the second stage is assumed to operate as a profit maximising monopolist in the telecom market, by choosing the optimal quantity he or she supplies. The realisation of Δ is assumed to be revealed right after the auction, or just before the winner has to make this decision.

3.1.2 The Auction

In the second stage the auctioneer runs a second-price royalty auction with a fixed-fee element (which will be formally defined below), in order to sell a single licence for operating the 3G technology in the market, for one period.

Two potential bidders participate in the auction. One is an incumbent who is assumed to be *risk-neutral*, and the other is a *risk-averse* entrant. This is a very important assumption in our model, and is the one that ultimately allows us to analyse the entry behaviour of non-incumbents, and to attribute any changes in that behaviour between different auction formats to their increased sensitivity to risky environments.

The entrant's utility function is assumed to take the form:

$$u(\pi) = 1 - e^{-\lambda\pi}$$

where π is profit and λ is the entrant's degree of (absolute) risk aversion.

Both bidders are assumed to have the same constant unit-cost of production $c \geq 0$. However, bidders' types are defined by their *fixed* costs of operating in the market.

We begin the analysis by establishing the simplest setting that still allows us to address the questions in the core of this study; for the moment we assume that the incumbent may have only one possible type. This means that the incumbent's fixed cost of production is set equal to I , moreover, we assume that this is common knowledge among both bidders and the auctioneer. The entrant's fixed cost of production E is a random variable with a rectangular distribution: $E \sim U [\underline{E}, \overline{E}]$, where $\underline{E} \leq I \leq \overline{E}$. The realised value of E is revealed to the entrant alone, only after he chooses 'entry' at the entry stage, and before he makes his bid in the auction.

The second-price royalty auction with a fixed-fee element is defined as follows; Bidders bid in the auction in terms of royalty rates satisfying $r \in [0, 1]$. The winner is

the bidder who bids the highest rate. Any bid made in the auction is a commitment by the bidder to pay the auctioneer a fixed fee of $F(r)$ right after the auction ends, and conditional upon winning. The auction follows the second-price format and thus r in the term $F(r)$ is the *second-highest* royalty rate that was bid in the auction (in what follows this will be referred to as 'the effective royalty rate'). $F(\cdot)$ is some function of the royalty rate that is set by the auctioneer. Recall that the effective royalty rate is also applied to the winner's revenue R (from operating in the telecom market for one period), to determine the amount of royalties he or she will have to pay (in addition to the amount $F(r)$) after the third stage of the game. The total payment the winner of the auction pays the auctioneer is therefore:

$$F(r) + rR$$

The function $F(\cdot)$ is designed by the auctioneer in advance of the auction, and is known to both bidders at the beginning of the game. At this point we place a single mild restriction on the possible shape of the $F(r)$ function - the total expected payoff of bidders from participating in the auction should be decreasing with the royalty rate, i.e. with their bids. This restriction is not too strong as it basically means that $F(r)$ should not decrease so fast that by increasing their royalty bids marginally, the loss the winner would make from the increase in the amount of royalties he or she will pay after the third stage, is more than offset by the gain from the drop in the cash payment after the auction. If this restriction does not hold bidders will have an incentive to increase their bid up to $r = 1$.

The auctioneer can also set a reservation royalty rate r_0 that is revealed to all bidders.

3.1.3 The Entry Stage

Entry to the telecom auction is costly - bidders in the auction are assumed to face a fixed cost of entry $f \geq 0$ which is not paid to the auctioneer or set by her. The entry cost f should be interpreted as the cost of actions such as conducting a market survey and preparing a business case, without which borrowing the money to finance an auction bid would be impossible. In telecom auctions these costs could be quite substantial, however, we assume, in accordance with real world evidence, that f is small enough so the incumbent will always enter, and thus we concentrate on the entrant's decision regarding whether or not to enter the auction.

Before the auction, and most importantly, before he learns the fixed cost E that determines his type, the entrant has to decide whether or not he wants to pay the cost of entry f and become a bidder in the auction. If he chooses to enter he then learns E and the second stage auction begins, if on the other hand he chooses not to enter, the incumbent wins the licence and pays the auctioneer: $F(r_0) + r_0R$.

3.2 Analysis of the Three-Stage Game

3.2.1 The Market

In the third stage of the game - the telecom market - there is no more uncertainty left unresolved. In particular, the value of the random variable Δ is realised, and so the market demand becomes known. Whoever wins the auction will therefore behave in the same way in the market; the owner of the licence operates as a profit maximising monopolist, whose profit is given by:

$$\pi(q) = [q(\frac{\Delta}{q})^{\frac{1}{2}} + p^0](1 - r) - cq - G$$

where the first term is the winner's revenue after paying the royalty rate r , and the second term is the variable cost, in the assumption of constant unit cost $c > 0$. The parameter $G \in \{I, E\}$ is the winner's fixed cost, I is the incumbent's fixed cost, and E is the entrant's. The fixed-fee component $F(r)$ of the winner's payment to the auctioneer, is assumed to be paid right after the auction ends, and before the winner begins to operate in the market and learns Δ . It is thus regarded as a sunk cost, and doesn't affect the winner's behaviour in the market.

Profit maximisation requires:

$$\begin{aligned} \frac{d\pi}{dq} &= (\frac{\Delta}{q})^{\frac{1}{2}}(1 - r) - \frac{1}{2}q(1 - r)(\frac{\Delta}{q})^{\frac{1}{2}}(\frac{1}{q}) - c = 0 \\ \frac{d\pi}{dq} &= \frac{1}{2}(1 - r)(\frac{\Delta}{q})^{\frac{1}{2}} - c = 0 \\ q^* &= (\frac{2c}{(1 - r)\Delta^{\frac{1}{2}}})^{-2} = \Delta(\frac{1 - r}{2c})^2 \end{aligned}$$

Substituting this into the inverse demand function to find the optimal price and revenue we get:

$$p^* = \left(\frac{\Delta}{q^*}\right)^{\frac{1}{2}} + \frac{p^0}{q^*} = \frac{2c}{1-r} + \frac{p^0}{\Delta} \left(\frac{2c}{1-r}\right)^2$$

$$R^* = \Delta \left(\frac{1-r}{2c}\right)^2 \left[\frac{2c}{1-r} + \frac{p^0}{\Delta} \left(\frac{1-r}{2c}\right)^2\right] = \Delta \left(\frac{1-r}{2c}\right) + p^0$$

where * represents optimal values.

It is now possible to find the maximal profit, which is a random variable before Δ is revealed. We call this random variable π_{\max} :

$$\begin{aligned} \pi_{\max} &= \left[\Delta \left(\frac{1-r}{2c}\right) + p^0\right](1-r) - c\Delta \left(\frac{1-r}{2c}\right)^2 - G \\ \pi_{\max} &= \Delta \frac{(1-r)^2}{2c} - \Delta \frac{(1-r)^2}{4c} + (1-r)p^0 - G \\ \pi_{\max} &= \Delta \frac{(1-r)^2}{4c} + (1-r)p^0 - G \end{aligned} \quad (1)$$

We can now try to say more about the magnitude of the constant p^0 that will be sufficient to insure that the winner of the auction does not make a loss in the telecom market. π_{\max} is an increasing function of Δ , which itself takes only non-negative values, is it therefore possible to insure that π_{\max} is always non-negative by imposing $\Delta = 0 \Leftrightarrow \pi_{\max} \geq 0$. This requires that:

$$p^0(1-r) \geq G$$

$$p^0 \geq \frac{G}{1-r}$$

where r is the effective royalty rate, and $G \in \{I, E\}$ is the winner's fixed cost.

It is already apparent that the minimum value of p^0 that is sufficient to insure that the operator of the licence in the telecom market breaks even is dependent on information - the effective royalty rate - that is unavailable to us before the auction stage.

3.2.2 The Auction

One licence, allowing its owner to sell telecom services to consumers for one period, is offered for sale using a second-price royalty auction with a fixed-fee element. At the time of the auction both bidders are uncertain as to the value of this licence; both, however, hold the same information with regards to its value. The information concerning bidders' types is not common though - the entrant alone knows his fixed cost is equal to E , while both bidders (and the auctioneer) are assumed to know that the incumbent's cost is I . The bidders' attitudes towards risk also affect their bidding behaviour. Since this auction is essentially a second-price auction, we assume both bidders will follow the simple dominant strategy of bidding their true valuation (in expectation) for the good. The risk-neutral incumbent will bid the royalty rate that equates her expected profit to zero, while the risk-averse entrant will bid the rate that equates his expected utility of profit to the utility he gets from zero profit. Recall however that the winner of the auction has to pay the auctioneer an additional amount of $F(r)$ right after the auction (where r is the second highest bid), and thus both bidders will take this into account when bidding. The entrant will therefore bid such

that:

$$Eu(\pi_{\max} - F(\rho^E)) = u(0)$$

while the incumbent will bid such that:

$$E(\pi_{\max} - F(\rho^I)) = 0$$

where ρ^E and ρ^I are the entrant's and incumbent's bids respectively.

The entrant's utility function is: $u(\pi) = 1 - e^{-\lambda\pi}$, to calculate his bid he will have to solve:

$$\begin{aligned} \int_0^\infty (1 - e^{-\lambda \frac{\Delta(1-\rho^E)^2}{4c}} e^{-\lambda(1-\rho^E)p^0} e^{\lambda E} e^{\lambda F(\rho^E)}) \frac{\Delta^m}{m!} e^{-\Delta} d\Delta &= 0 \\ 1 - e^{-\lambda(1-\rho^E)p^0} e^{\lambda E} e^{\lambda F(\rho^E)} \int_0^\infty e^{-\lambda \frac{\Delta(1-\rho^E)^2}{4c}} \frac{\Delta^m}{m!} e^{-\Delta} d\Delta &= 0 \end{aligned}$$

Define: $\Delta(\frac{\lambda(1-\rho^E)^2}{4c} + 1) = \Delta\beta = x$. The entrant's problem then becomes:

$$\begin{aligned} 1 - e^{-\lambda(1-\rho^E)p^0} e^{\lambda E} e^{\lambda F(\rho^E)} \int_0^\infty e^{-x} \left(\frac{x}{\beta}\right)^m \left(\frac{1}{m!}\right) \frac{dx}{\beta} &= 0 \\ 1 - e^{-\lambda(1-\rho^E)p^0} e^{\lambda E} e^{\lambda F(\rho^E)} \left(\frac{1}{\beta}\right)^{m+1} \left(\frac{1}{m!}\right) \int_0^\infty e^{-x} x^m dx &= 0 \end{aligned}$$

The integral can be evaluated by iterated integration by parts:

$$\int_0^\infty e^{-x} x^m dx = m!$$

Substituting the definition of β into what is left we get:

$$1 - e^{-\lambda(1-\rho^E)p^0} e^{\lambda E} e^{\lambda F(\rho^E)} \left(\frac{\lambda(1-\rho^E)^2}{4c} + 1\right)^{-(m+1)} = 0$$

Taking the logarithm of both sides we can define the entrant's bidding function:

$$0 = \lambda E + \lambda F(\rho^E) - \lambda(1 - \rho^E)p^0 - (m+1) \ln\left(1 + \frac{\lambda(1 - \rho^E)^2}{4c}\right)$$

$$E + F(\rho^E) = p^0(1 - \rho^E) + \frac{m+1}{\lambda} \ln\left(1 + \frac{\lambda(1 - \rho^E)^2}{4c}\right)$$

Assuming λ is positive but small, we can use a Taylor series approximation of the last term, in which terms in λ^3 and higher are neglected, to simplify further the formula for the entrant's bidding function:

$$E + F(\rho^E) = p^0(1 - \rho^E) + \frac{m+1}{\lambda} \left[\frac{\lambda(1 - \rho^E)^2}{4c} - \frac{1}{2} \frac{\lambda^2(1 - \rho^E)^4}{16c^2} \right]$$

$$E + F(\rho^E) = p^0(1 - \rho^E) + \frac{(m+1)}{4c} (1 - \rho^E)^2 - \frac{\lambda(m+1)}{32c^2} (1 - \rho^E)^4 \quad (2)$$

A similar calculation could be carried out in order to get to the incumbent's bidding function:

$$\int_0^\infty \left(\Delta \frac{(1 - \rho^I)^2}{4c} + p^0(1 - \rho^I) - I - F(\rho^I) \right) \frac{\Delta^m}{m!} e^{-\Delta} d\Delta = 0$$

$$p^0(1 - \rho^I) - I - F(\rho^I) + \int_0^\infty \Delta \frac{(1 - \rho^I)^2}{4c} \frac{\Delta^m}{m!} e^{-\Delta} d\Delta = 0$$

$$p^0(1 - \rho^I) - I - F(\rho^I) + \frac{(1 - \rho^I)^2}{m!4c} \int_0^\infty \Delta^{m+1} e^{-\Delta} d\Delta = 0$$

Integrating by parts:

$$\int_0^\infty \Delta^{m+1} e^{-\Delta} d\Delta = (m+1)!$$

We get the incumbent's bidding function:

$$I + F(\rho^I) = p^0(1 - \rho^I) + \frac{(m+1)}{4c} (1 - \rho^I)^2 \quad (3)$$

Note that, as expected, the incumbent's bidding function (3) is the same as that of the entrant's (2), with $\lambda = 0$.

We can now examine the two bidding functions in order to identify the forces that influence the bidding, and could be used by the auctioneer to achieve her goals. The LHS of both (2) and (3) represent the bidders' fixed-fee component of the payment, and it is clear from the formula that whether this payment is made to the auctioneer after the auction $F(\cdot)$, or it is invested in the building of the infrastructure at the beginning of the third stage, makes no difference to the bidders, and both will react in the same way to a change in either of them. Bidders will differ however in the magnitude of their reaction to a change in the fixed-fee component, with the entrant reacting more rapidly to such a change.

These will turn out to be important observations, as the design of the fixed fee schedule $F(r)$ is the main tool in the hands of the auctioneer in this framework, which she will use in order to manipulate the bidding to achieve her goals.

3.2.3 The Entry Stage

At the first stage of the game the entrant is the only one who moves - he has to decide whether or not he is willing to pay the cost f of entry in order to become a bidder in the auction. When he makes this decision the entrant does not know his own type E yet. Otherwise, his information is similar to that of the incumbent (and the auctioneer) at this stage. It includes: the incumbent's type I ; the distribution of the entrant's type $E \sim U[\underline{E}, \overline{E}]$; the entry cost f ; and the exact form of the fixed-fee

function $F(r)$. The entrant will use all of the information in his possession in his decision regarding entry.

We assume the boundaries of the distribution of the entrant's type \underline{E} and \bar{E} are such that there exists a value for the entrant's fixed cost for which a tie in the bidding occurs. We call this value $E_{tie} : E = E_{tie} \implies \rho^E = \rho^I$. It is clear that the entrant will win the auction whenever $E \leq E_{tie}$, and will lose it otherwise. Since the entrant is risk-averse while the incumbent is risk-neutral, we expect that: $E_{tie} \leq I$. The value of E_{tie} can be calculated using (2) and (3) to verify this relationship:

$$E_{tie} + F(\rho^{E_{tie}}) = p^0(1 - \rho^{E_{tie}}) + \frac{(m+1)(1 - \rho^{E_{tie}})^2}{4c} - \frac{\lambda(m+1)(1 - \rho^{E_{tie}})^4}{32c^2}$$

But by definition we have: $\rho^{E_{tie}} = \rho^I$, and so from (3) we get:

$$E_{tie} = I - \frac{\lambda(m+1)(1 - \rho^I)^4}{32c^2} \quad (4)$$

The value of E_{tie} turns out to be a good measure of the entrant's *relative* position in the auction, and thus central in the analysis of the entrant's entry decisions, with higher values corresponding to more chances of entry. We therefore turn to a careful examination of the nature of E_{tie} .

It is apparent from (4) that E_{tie} is a function of the incumbent's known type I and her bid ρ^I , which itself is dependent on her type I and on the fixed fee $F(\rho^I)$ that corresponds to her bid. It is independent of the entrant's actual type E . It is interesting to note that while an increase in either I or F^7 will affect ρ^I in the same way - both will cause the incumbent to reduce her bid - they have a very different

⁷ F here is constant to indicate the value of the fixed fee at the incumbent's bid.

effect on E_{tie} , with the former causing it to increase and the latter causing it to drop. In other words, for a given $F(\cdot)$ function, a drop in the incumbent's bid ρ^I could only be a result of an increase in her fixed cost I , this is a piece of good news for the entrant since the incumbent's cost has no bearing on him, his *relative* position in the auction is improved, and the increase in E_{tie} is an expression of this. This effect is depicted in figure 3. On the other hand, for a given known value for the incumbent's cost I , a similar drop in her bid is a signal of an increase in the fixed fee. This piece of news is not such a good one for the entrant as the fixed fee applies for both bidders and thus *he* reduces his bid as a result as well; moreover the entrant does so by more than the incumbent (due to his risk-aversion) and so his *relative* position in the auction worsens. This effect is depicted in figure 4.

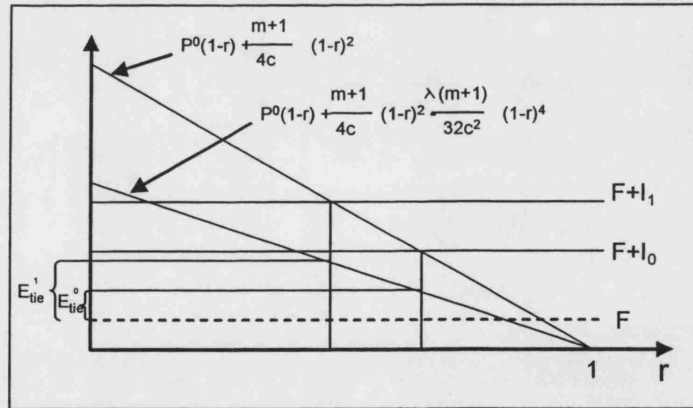


Figure 3: E_{tie} is increasing with I .

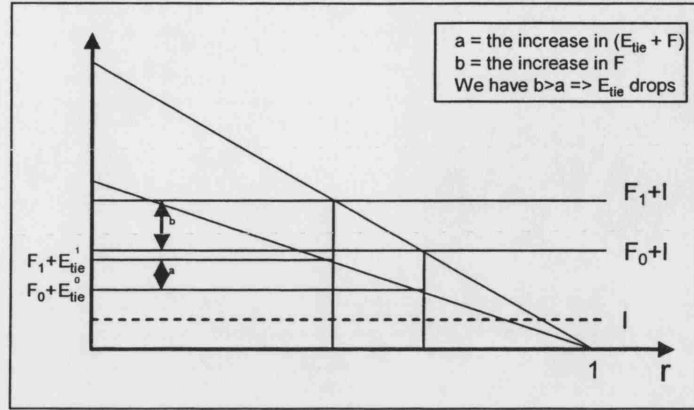


Figure 4: E_{tie} is decreasing with F .

Note that the fact that the figures are drawn with a constant $F(r)$ function is not meant to prejudice the optimal shape of the fixed-fee schedule. We will examine the auctioneer's task in designing the full schedule in the next chapter.

The entrant will decide, in the first stage of the game, to enter the auction whenever the expected utility he derives from bidding in the auction is greater than or equal to the utility he gets from staying out. That is, he enters whenever the following expression is non-negative:

$$\int_{\underline{E}}^{E_{tie}} (1 - e^{\lambda f} e^{\lambda E} e^{\lambda F(\rho^I)} e^{-\lambda p^0(1-\rho^I)} (\frac{\lambda(1-\rho^I)^2}{4c} + 1)^{-(m+1)}) f(E) dE + \int_{E_{tie}}^{\bar{E}} (1 - e^{\lambda f}) f(E) dE \quad (5)$$

The first integral in (5) is the expectation of the entrant's utility if he enters the auction and wins, the second integral is his expected utility if he enters and loses, this has to be equal to or greater than zero, which is what the entrant gets if he decides not to

participate in the auction at all. Note that the royalty rate that appears in (5) is the incumbent's bid ρ^I , which will be the effective royalty rate in case the entrant wins the auction, and which is assumed at this point to be known to the entrant (we still assume that I is common knowledge at this point).

Recall that the entrant does not know his own fixed cost at this stage and so his decision regarding entry is based on his expected position in the bidding competition, relative to that of the incumbent, which he knows. The incumbent's bid, see equation (3), is a function of her fixed cost I and the value of the fixed fee at her bid $F(\rho^I)$, this is all the information the entrant needs in order to make his entry decision, (except for the entry cost f).

Let us characterise the entrant's decision whether or not to enter the auction as follows; for a given cost of entry f , there exists a value of royalty rate, so that, *if circumstances were to lead the incumbent to bid exactly that value*, the entrant will find himself indifferent with regard to entering the auction. We call this value $r^*(I, F(\rho^I), f)$, and it is defined by equating the expression in (5) to zero, replacing the incumbent's bid ρ^I with r^* and solving. It is not necessary, however, to solve explicitly for the threshold value r^* in order to analyse some of its characteristics.

It is important to note that the threshold value r^* is defined *for a given set of variables*: $(I, F(\rho^I), f)$ and that there is only a single point on the fixed-fee schedule that affects this value - the value of the fixed fee at the incumbent's bid⁸. The value

⁸ Recall that at the time he decides on entry the entrant does not know his own cost. All the information he has at this point is epitomised in the incumbent's bid which is affected by only a single value of fixed fee. This point is further explained below.

at which the entrant becomes indifferent to entry is a measure of how favourable for him are these exogenous variables, with higher values of r^* indicating better conditions entry-wise. The value of the incumbent's cost I is set by nature before any interaction begins, and is thus the basic exogenous variable of the model over which no player has any power. In this chapter, we assume that this value is common knowledge among all players, and thus the auctioneer can calculate what would be the incumbent's bid, and the threshold value of it for every level of fixed fee.

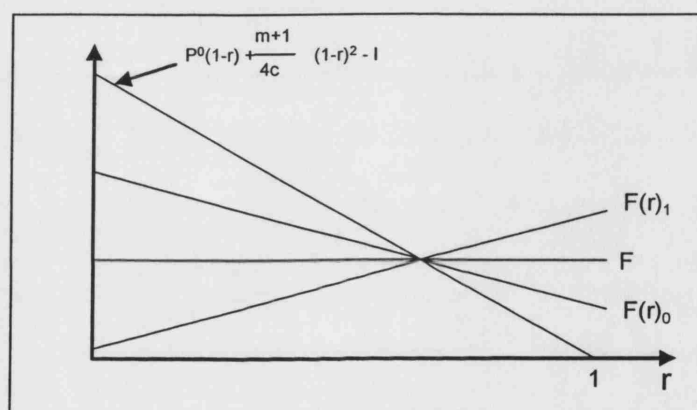


Figure 5: Different fixed-fee schedules.

From Figure 5 it is clear that the incumbent will bid the same rate of royalties as long as the fixed fee crosses her payoff curve at the same point, and that no other point on the fixed-fee schedule has any effect on her behaviour. Given that the entrant does not know his own cost at the entry stage, and that the same fixed-fee schedule applies to both bidders, the entrant too would make the same entry decision (but not bid) whichever of these schedules is chosen. We can therefore refer to all of the

fixed-fee schedules in Figure 5 as F - the fixed fee at the point of intersection with the incumbent's payoff curve (minus the known I) - and thus create a one-to-one relationship between the auctioneer's choice of a fixed fee (F) to the incumbent's bid (ρ^I) which determines the entrant's entry decision. For ease of notation we can now neglect the entry cost f and express the threshold royalty rate as $r_I^*(F)$, for the purpose of examining the nature of the relationship that it captures.

We now want to analyse the threshold value for the incumbent's bid $r_I^*(F)$ for different values of fixed fee, and look at the information captured by this value. Firstly we note again that a higher value of $r_I^*(F)$ is an indication of better conditions entry-wise, that is, if the entrant becomes indifferent to entry at a high value for the effective royalty rate, this means that the auction on the whole seems like a rather good investment, and only if he has to pay a very significant share of his revenue to the auctioneer will the entrant become indifferent and eventually opposed to entry. Secondly, for any given set (I, F) the incumbent chooses her bid $\rho^I(F)$, and the entrant can calculate the threshold value $r_I^*(F)$, the two can now be compared; if we find that $\rho^I(F) > r_I^*(F)$ this means that the fixed fee F is too high for the entrant to enter, while if $\rho^I(F) \leq r_I^*(F)$ this means that the entrant is willing to pay a higher fixed fee and still enter. There is only a *single* level of fixed fee that will equate the two and cause the entrant to be just indifferent to entry, for the given known value of I .

This relationship may seem somewhat confusing - if the actual incumbent's bid is below the threshold value the entrant will choose to enter, while if it lies above the threshold value no entry will occur - but the key to its understanding is to remember that we are holding I constant, and think of the case of no fixed fee, i.e. $F = 0$. With no fixed fee we are at the 'pure royalty' case (in the terminology of the first chapter) in which the entrant's disadvantage (from being more risk-averse) is absent. In the 'pure royalty' case we therefore assume that the entrant always chooses entry⁹, and the threshold value is equal to one; $r_I^*(0) = 1$. Now consider the auctioneer gradually increasing the fixed fee from zero - both bidders become worse-off and reduce their bids, but the entrant does so faster than the incumbent (see figure 4) and so his *relative* position (for which $r_I^*(F)$ is a measure) also worsens. The threshold value $r_I^*(F)$ drops as a result of the increase in F . We know that eventually, if the fixed fee is increased enough, there *must* come a point where it is not worthwhile for the entrant to enter anymore. At the exact point when that happens, *by definition* we must have: $\rho^I(F) = r_I^*(F)$. From this we can reduce that the threshold value $r_I^*(F)$ moves in the same direction as $\rho^I(F)$, but at a faster rate, as a result of an increase in F . Moreover, as long as the fixed fee has not yet reached a level high enough for the entrant to change his mind about entry we would still have: $\rho^I(F) < r_I^*(F)$, and if, on the other hand, the fixed fee continues to increase beyond the point of

⁹ If the incumbent's cost is so low that even in the 'pure royalty' case the entrant does not find it worth while to enter, then there is nothing the auctioneer can do via the the design of the auction to change that, as she has no influence over bidders' costs. We therefore assume that the incumbent's cost is not 'too low' in that sense, as we see little point in discussing entry in a setting where it could never occur.

indifference for the entrant, we would see: $\rho^I(F) > r_I^*(F)$. See Figure 6 for the graphical representation.

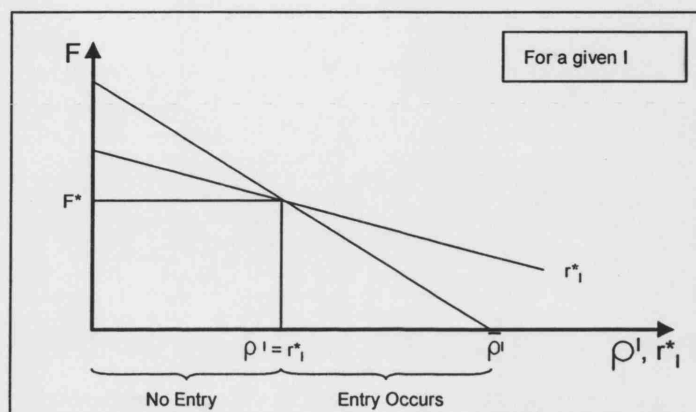


Figure 6: $r_I^*(F)$ drops as F increases.

The threshold royalty rate turns out to be at the center of the auctioneer's task of designing the optimal second-price royalty auction with a fixed-fee element, and so we will return to a fuller analysis of it in the next chapter, for the moment however it suffices to note that the auctioneer can influence the entrant's decision with regards to entry through the incumbent's bid, via the way she sets the fixed fee.

3.3 Do Royalties Promote Entry?

In this section we isolate the effect of the format of the second-price royalty auction with a fixed-fee element on an entrant's view of the auction as a potential investment. The optimal design of this auction is yet to be addressed, we are interested in

investigating the effect of the mere use of royalty bidding on a risk-averse potential entrant.

For this purpose we use the definition of E_{tie} (see equation (4)) as a measure of entry, with higher values corresponding to higher chances of entry. We will compare E_{tie} with an equivalent measure, created for an equivalent environment, where the only difference is the auction format.

We now analyse a three-stage game in which all the assumptions of this chapter remain intact, except that the second stage is a regular, cash, second-price auction. We let the same pair of bidders - a risk-neutral incumbent and a risk-averse entrant - choose their actions at every stage, but we will be paying attention in particular to the entrant's decision in the entry stage before the auction, while all other aspects will be presented in a brief manner.

3.3.1 The Market

Market demand is assumed as before to take the form: $p = (\frac{\Delta}{q})^{\frac{1}{2}} + \frac{p^0}{q}$ where Δ follows a Gamma distribution with pdf $f(\Delta) = \frac{1}{m!} \Delta^m e^{-\Delta}$. The value of Δ is realised after the auction, so that the winner operates in a market with no uncertainty, as a profit maximising monopolist, for one period. His or her profit at the end of this period is:

$$\pi(q) = q[(\frac{\Delta}{q})^{\frac{1}{2}} + \frac{p^0}{q}] - qc - G - B$$

where the first term is the winner's revenue, c is the constant unit-cost, $G \in \{I, E\}$ is the winner's fixed cost, and B is the second-highest cash bid made in the auction, i.e. B is the amount the winner pays the auctioneer.

Profit maximisation requires:

$$\begin{aligned}\frac{d\pi}{dq} &= \frac{1}{2}\left(\frac{\Delta}{q}\right)^{\frac{1}{2}} - c = 0 \\ \left(\frac{\Delta}{q}\right)^{\frac{1}{2}} &= 2c\end{aligned}$$

Optimal values are given by:

$$\begin{aligned}q^* &= \frac{\Delta}{4c^2} \\ p^* &= 2c + \frac{4c^2}{\Delta}p^0 \\ R^* &= \frac{\Delta}{2c} + p^0\end{aligned}$$

We can now formulate the variable π_{\max} as before:

$$\pi_{\max} = \frac{\Delta}{4c} + p^0 - B - G$$

3.3.2 The Auction

A licence for operating in the market analysed above is offered on sale using a regular, cash, second-price auction. Two bidders participate; a risk-neutral incumbent, whose fixed cost is commonly known to equal I , and a risk-averse entrant whose fixed cost E follows $E \sim U[\underline{E}, \bar{E}]$, and is revealed to him just before the auction starts. The entrant's utility function is again assumed to be $u(\pi) = 1 - e^{-\lambda\pi}$, and so he will bid

the amount B_E that solves:

$$Eu(\pi_{\max}) = u(0)$$

$$Eu(\pi_{\max}) = \int_0^\infty (1 - e^{-\lambda \frac{\Delta}{4c}} e^{-\lambda p^0} e^{\lambda B_E} e^{\lambda E}) \frac{\Delta^m}{m!} e^{-\Delta} d\Delta = 0$$

$$Eu(\pi_{\max}) = 1 - e^{-\lambda p^0} e^{\lambda B_E} e^{\lambda E} \int_0^\infty e^{-\lambda \frac{\Delta}{4c}} e^{-\Delta} \frac{\Delta^m}{m!} d\Delta = 0$$

Define: $\Delta(\frac{\lambda}{4c} + 1) = \Delta\beta = x$, and substitute in to get:

$$1 - e^{-\lambda p^0} e^{\lambda B_E} e^{\lambda E} \frac{1}{m!} \int_0^\infty e^{-x} \left(\frac{x}{\beta}\right)^m \frac{dx}{\beta} = 0$$

$$1 - e^{-\lambda p^0} e^{\lambda B_E} e^{\lambda E} \frac{1}{m!} \left(\frac{1}{\beta}\right)^{m+1} \int_0^\infty e^{-x} x^m dx = 0$$

Integration by parts yields: $\int_0^\infty e^{-x} x^m dx = m!$, and so we can get:

$$1 - e^{-\lambda p^0} e^{\lambda B_E} e^{\lambda E} e^{-(m+1) \ln(1 + \frac{\lambda}{4c})} = 0$$

Taking logarithm, and dividing by λ :

$$E + B_E = p^0 + \frac{m+1}{\lambda} \ln(1 + \frac{\lambda}{4c})$$

We can once again simplify this using a Taylor series approximation, neglecting terms in λ^3 , to get to the entrant's bidding function:

$$\begin{aligned} E + B_E &= p^0 + \frac{m+1}{\lambda} \left(\frac{\lambda}{4c} - \frac{1}{2} \frac{\lambda^2}{16c^2} \right) \\ B_E &= p^0 + \frac{m+1}{4c} - \frac{\lambda(m+1)}{32c^2} - E \end{aligned} \quad (6)$$

Similarly we can formulate the incumbent's bidding function by solving:

$$\begin{aligned} E\pi_{\max} &= \int_0^\infty \left(\frac{\Delta}{4c} + p^0 - I - B_I \right) \frac{\Delta^m}{m!} e^{-\Delta} d\Delta = 0 \\ E\pi_{\max} &= p^0 - I - B_I + \frac{1}{m!4c} \int_0^\infty \Delta^{m+1} e^{-\Delta} d\Delta = 0 \end{aligned}$$

Integration by parts yields: $\int_0^\infty \Delta^{m+1} e^{-\Delta} d\Delta = (m+1)!$, and so the incumbent's bidding function takes the form:

$$B_I = p^0 + \frac{m+1}{4c} - I \quad (7)$$

3.3.3 The Entry Stage

In this stage the entrant decides whether or not he wants to pay the cost of entry f , and become a bidder in the auction. We also maintain the important assumption that the entrant does not know his type when he decides on entry.

Applying the same argument as before, we assume that there exists a value for the entrant's fixed cost such that a tie in the bidding occurs. We call this value E_{tie}^B , so that: $E = E_{tie}^B \implies B_E = B_I$. The entrant will win the auction whenever his fixed cost is lower than E_{tie}^B , and lose otherwise. E_{tie}^B can be calculated using both bidders' bidding functions (6) and (7):

$$E_{tie}^B = p^0 + \frac{m+1}{4c} - \frac{\lambda(m+1)}{32c^2} - B_{E_{tie}^B}$$

By definition we have: $B_{E_{tie}^B} = B_I$, substituting this in we get:

$$\begin{aligned} E_{tie}^B &= p^0 + \frac{m+1}{4c} - \frac{\lambda(m+1)}{32c^2} + I - p^0 - \frac{m+1}{4c} \\ E_{tie}^B &= I - \frac{\lambda(m+1)}{32c^2} \end{aligned} \quad (8)$$

We are now in a position to compare the two values for the entrant's fixed cost that would cause a tie in the bidding, in the two alternative auction formats. As everything but the format of the auction in the second stage is constant between the

two games, these values are sufficient to compare the likelihood of entry occurring in the two auctions, with higher values corresponding to higher chances of entry.

In the second-price royalty auction with a fixed fee element we have:

$$E_{tie} = I - \frac{\lambda(m+1)(1-\rho^I)^4}{32c^2} \quad (4)$$

While in the second-price cash auction we have:

$$E_{tie}^B = I - \frac{\lambda(m+1)}{32c^2} \quad (8)$$

The comparison of equations (4) and (8) makes it clear that for all non-negative bids of the incumbent the second-price auction with a fixed-fee element performs just as well or better on entry, than the second-price cash auction, i.e. for all $\rho^I \geq 0$ we have $E_{tie} \geq E_{tie}^B$.

The intuition behind this result is simple, if everything else is constant, the bid made by the risk-averse entrant is higher, *relative to that of the incumbent*, the higher is the royalty rate payable by the winner. Moreover, different values of fixed fee that are added to the winner's payment may change the way both bidders bid, but as long as the auctioneer makes sure that they have an incentive to bid a positive rate, this effect remains.

We can now summarise this first result:

Proposition 1 *Other things being equal, a risk-averse entrant is more likely to participate in a second-price royalty auction with a fixed-fee element, than in a second-price cash auction.*

This conclusion could be compared to a known result from the literature on optimal incentive contracts. It has been shown for example in McAfee & McMillan (1986), that including a royalty component in the winner's payment serves to increase the bidding competition among bidders, so that all bidders bid higher. The authors also show that with (heterogeneous) risk-averse bidders, the optimal level of (pre-set) royalty rate is increasing in the degree of risk aversion the bidders exhibit. However, McAfee & McMillan (1986) is concerned mainly, (as indeed most of the literature on auctions with royalties), with ways the auctioneer can extract more informational rents from bidders in order to increase her revenue from the auction. We on the other hand, are looking for ways to make the auction more attractive to weak bidders relative to their stronger counterparts. In fact we will show below that the introduction of royalties in our setting does not serve to increase the auctioneer's revenue at all.

3.4 Revenue and Entry

In this section we turn to the question of optimal design of the second-price auction with a fixed-fee element. By the 'optimal design' we mean, as is usually the case, an implementation the auction that generates an outcome that reflects the auctioneer's preferences as closely as possible. However, the preferences we attribute to the auctioneer, take a different form than that typically used. We assume that the telecom auctioneer cares directly about entry into the auction, and not about revenue alone. In the first chapter we elaborated on the unique telecom auction environment, where

in the move from beauty contests and lotteries to auctions, a real and active bidding competition, in which winners had to ‘prove’ their superiority to their opponents, became crucial to the auction’s perceived (and indeed sometimes actual) success. We therefore built entry into the auctioneer’s welfare function, which will be defined below as a weighted sum of entry and revenue.

In the last section we established that for a given I , higher royalty bids made by the incumbent correspond to a lower level of fixed fee, and are thus an indication of favourable conditions for the entrant, i.e. that in our setting a lower fixed fee promotes entry. We would now like to analyse the auctioneer’s task of setting the fixed fee optimally, and its overall effect on other components of the auctioneer’s objective function. For this purpose we need to say more about the nature of the auctioneer’s preferences. For the moment we make the assumption that the auctioneer cares both about entry and about revenues in the sense that her welfare function is a weighted sum of the two without being specific about the weights¹⁰.

¹⁰ The auctioneer’s revenue appears in her welfare function (rather than say consumer surplus) for three reasons; Firstly, we want to be able to measure the ‘cost’ to the auctioneer of promoting entry by comparing revenues of different auction formats. Secondly, practical experience shows us that telecom auctioneers do care about their revenues, both on its own and as an indication of efficiency. And thirdly, even in a simplified setting, the inclusion of consumer surplus in the auctioneer’s objective function turn out to considerably complicate the calculations.

3.4.1 The Auctioneer's Revenue

In order to compute the auctioneer's revenue from the second-price royalty auction with a fixed-fee element, in its current setting, we have to consider two possible scenarios:

1. Only the incumbent enters the auction.
2. Both the incumbent and the entrant enter.

The auctioneer's expected revenue under the first scenario is:

$$EAR_1 = F(r_0) + r_0[(m+1)(\frac{1-r_0}{2c}) + p^0]$$

where r_0 is the reservation royalty rate, $(m+1)$ is the expected value of Δ , and the term in the square brackets is the incumbent's optimal revenue.

In the simple setting we are currently considering (where the incumbent's type is common knowledge), the optimal value for the reservation royalty rate becomes immediately apparent. Since the auctioneer is assumed to know the value of I in advance, she can easily calculate what the incumbent's bid will be, and set the reservation rate accordingly, i.e. she should set $r_0 = \rho^I$. However, advertising this reservation royalty rate in the beginning of the game, might affect the entrant's decision on entry. It is therefore in the auctioneer's interest to conceal the value of r_0 until the end of the bidding, where bids which are lower than the reservation are disregarded. This policy is compatible with the bidders' strategies; the incumbent has no incen-

tive to deviate from bidding the rate that reflects her true valuation. As to the entrant, if he chooses to stay out, there is no reason for him to change that decision in light of the reservation royalty rate, and if he chooses to enter the auction, he either wins, in which case he anyway expects the effective royalty rate to equal the incumbent's bid, or he loses in which case his bid is lower than that of the incumbent's and therefore is disregarded at the end of the auction. In any case he has no incentive to deviate.

After setting the reservation royalty rate we would then have¹¹:

$$EAR_1 = F(\rho^I) + \rho^I[(m+1)(\frac{1-\rho^I}{2c}) + p^0]$$

The auctioneer's expected revenue under the second scenario, where the entrant chooses to enter the auction, is:

$$EAR_2 = \int_{\underline{E}}^{E_{tie}} \{F(\rho^I) + \rho^I[(m+1)(\frac{1-\rho^I}{2c}) + p^0]\} \frac{1}{\bar{E} - \underline{E}} dE + \int_{E_{tie}}^{\bar{E}} \{F(\rho^E) + \rho^E[(m+1)(\frac{1-\rho^E}{2c}) + p^0]\} \frac{1}{\bar{E} - \underline{E}} dE$$

where the first integral is the expected revenue of the auctioneer if the entrant wins the auction, and the second integral is her expected revenue if the incumbent wins.

The first integrand is independent of E and so we have:

$$EAR_2 = \frac{E_{tie} - \underline{E}}{\bar{E} - \underline{E}} \{F(\rho^I) + \rho^I[(m+1)(\frac{1-\rho^I}{2c}) + p^0]\} + \int_{E_{tie}}^{\bar{E}} \{F(\rho^E) + \rho^E[(m+1)(\frac{1-\rho^E}{2c}) + p^0]\} \frac{1}{\bar{E} - \underline{E}} dE$$

¹¹ Obviously the assumption regarding the auctioneer's knowledge of the incumbent's fixed cost is very strong and not very realistic - a fuller setting with a more realistic assumption about the structure of information is considered in the next chapter.

Taking account of the fact that the effective royalty rate can never be less than the reservation royalty rate (which is equal to the incumbent's bid), and of that when the entrant loses the auction this means that he bids a lower rate than the incumbent, and his bid is therefore disregarded and replaced by the reservation rate, we can simplify further and express the auctioneer's revenue in terms of the incumbent's bid alone:

$$EAR_2 = F(\rho^I) + \rho^I[(m+1)(\frac{1-\rho^I}{2c}) + p^0]$$

It is now possible to join the two scenarios together to get the overall auctioneer's revenue as a function of the incumbent's bid:

$$AR = F(\rho^I) + \rho^I[(m+1)(\frac{1-\rho^I}{2c}) + p^0]$$

We now want to make use of the fact that was already presented above that for a given value of I the incumbent, when choosing her bid, cares only about the point of intersection between her own payoff curve and the fixed-fee schedule. We therefore noted above that it is useful to 'label' the fixed-fee schedule according to its level at this point of intersection F , which is actually equivalent to dealing with a constant schedule. Moreover, for a given I there exists a one-to-one relationship between the fixed fee F and the incumbent's bid ρ^I . We therefore can express the auctioneer's revenue as a function of $\rho^I(F)$:

$$AR(\rho^I(F)) = F + \rho^I(F)[(m+1)(\frac{1-\rho^I(F)}{2c}) + p^0]$$

From equation (3) it is possible to substitute for F :

$$\begin{aligned}
AR(\rho^I(F)) &= p^0(1 - \rho^I(F)) + (m+1)\frac{(1 - \rho^I(F))^2}{4c} - I + \\
&\quad (m+1)\rho^I(F)\frac{1 - \rho^I(F)}{2c} + \rho^I(F)p^0 \\
AR(\rho^I(F)) &= p^0 + \frac{m+1}{4c}[(1 - \rho^I(F))^2 + 2(\rho^I(F) - \rho^I(F)^2)] - I \\
AR(\rho^I(F)) &= p^0 + \frac{m+1}{4c}(1 - \rho^I(F)^2) - I \tag{9}
\end{aligned}$$

It is clear from (9) that the auctioneer's revenue in this setting is maximised when the fixed fee is set such that the incumbent finds it in her interest to bid $\rho^I = 0$. We know that (for a given I) the incumbent reduces her bid as a result of an increase in the fixed fee.

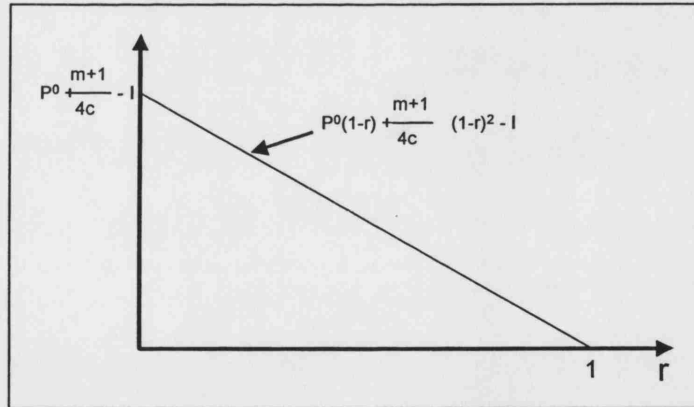


Figure 7: ρ^I drops as F increases.

Figure 7 shows the bidding strategy for the incumbent. It is clear from the figure that she will find it in her interest to actually bid a zero royalty rate only if the

fixed fee she has to pay is greater than:

$$p^0 + \frac{(m+1)}{4c} - I$$

But this is in fact exactly equal to the incumbent's expected profit from the second-price auction with no royalty rate (see equation (1) with $r = 0$ and equation (7)), i.e. this would be the incumbent's bid in a second-price *cash* auction.

3.4.2 Revenue and Entry - a trade-off

The observation made in the previous paragraph leads to an interesting feature of the second-price royalty auction with a fixed-fee element in this setting - the introduction of royalty bidding, does promote entry but at the expense of reducing the auctioneer's revenue. The auctioneer's revenue is actually maximised with a zero royalty rate, when it also coincides with the revenue from the second-price cash auction.

This feature introduces a trade-off between revenue and entry in designing the fixed-fee schedule, and it makes the second-price cash auction a special case of the second-price royalty auction with a fixed-fee element, where $F = p^0 + \frac{(m+1)}{4c} - I$.

The auctioneer's revenue is dependent, in this setting, only on the incumbent's bid ρ^I , which in turn depends (for a given I) only on the fixed fee at a certain point, that is $F(\rho^I) = F$. We can express entry in terms of the incumbent's bid as well, using the threshold value r_I^* - the value of the incumbent's bid that will cause the entrant to be just indifferent to entry. There is a single value of fixed fee that, if set by the auctioneer, will cause the incumbent to bid exactly this threshold value, call

this value F^* . All levels of fee which are higher than F^* will cause the entrant not to enter and thus we will have: $\rho^I(F) > r_I^*(F)$, fees lower than F^* will induce entry and thus: $\rho^I(F) \leq r_I^*(F)$. We can now define the entry function as follows:

$$g(\rho^I(F)) = \begin{cases} 0 & \text{if } \rho^I(F) > r_I^*(F) \\ 1 & \text{if } \rho^I(F) \leq r_I^*(F) \end{cases}$$

And the auctioneer's welfare function thus take the form:

$$W(\rho^I(F)) = \alpha AR(\rho^I(F)) + \beta g(\rho^I(F))$$

where α and β are the weights the auctioneer puts on revenue and entry respectively. Figure 8a shows the functions $AR(\cdot)$ and $g(\cdot)$, and Figure 8b shows the auctioneer's welfare function for the case of equal weights $\alpha = \beta = 1$. (Since p^0 is a positive constant it does not affect the qualitative demonstration and thus is omitted from the figure).

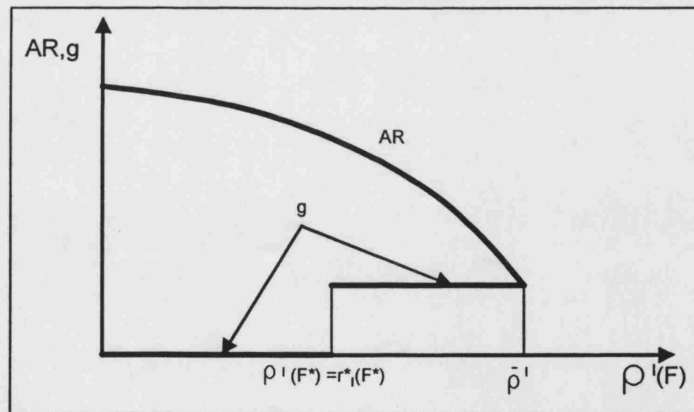


Figure 8a: The functions $AR(\rho^I)$ and $g(\rho^I)$.

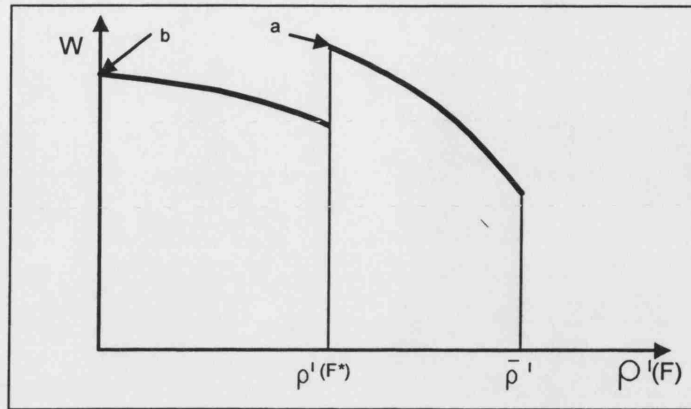


Figure 8b: The welfare function $W(\rho^I(F))$.

In the figures $\bar{\rho}^I$ is the highest possible bid for the incumbent - the bid she makes with $F = 0$. Note that in order to move along the auctioneer's welfare function, from right to left say, the auctioneer needs to increase gradually the level of the constant fixed fee, when this level reaches F^* the entrant will become just indifferent to entry, and from that point on will not enter the auction. See Figure 6 for a graphical demonstration of how the switch point at the entrant's indifferent value is determined. Note also that, contrary to what may seem intuitive, on the right of the point where the incumbent's bid coincides with its threshold value, (i.e. for $F < F^*$) we have $\rho^I(F) < r_I^*(F)$, while on the left of it (i.e. with $F > F^*$) we have $\rho^I(F) > r_I^*(F)$. This is an expression of the entrant's risk-aversion which makes him more sensitive to changes in the fixed fee, and thus he reduces his bid more rapidly than the incumbent as a result of an increase in F , so that his relative position worsens as well. The

drop in the entrant's indifferent value $r_I^*(F)$ as a result of an increase in the fixed fee is an expression of the conditions in the auction becoming less favourable to him, as the lower is the rate at which the entrant becomes indifferent to entry, the worse does the auction seem as an investment opportunity.

3.4.3 The Optimal Fixed Fee

The task of optimally designing the second-price royalty auction with a fixed-fee element is now reduced to choosing a single value for the fixed fee. Since the auctioneer is assumed to know the incumbent's fixed cost in advance there is a single value on the fixed-fee schedule that affects entry. Setting a single value of fixed fee is equivalent to dealing with a constant schedule for the fixed fee. In the next chapter we deal with multiple incumbent types and thus the task of designing the optimal fixed fee will involve a complete schedule.

We know that the auctioneer will never have an incentive to set the fixed fee lower than F^* - the fee that unites the incumbent's bid $\rho^I(F^*)$ with the threshold value $r_I^*(F^*)$ - since it would then be possible for her to increase her revenue without hurting entry, by increasing marginally the fixed fee until it reaches F^* . Considering only values of F that induce the incumbent to bid $\rho^I(F) \geq r_I^*(F)$ it is clear from looking at Figure 8b that there are only two values that are candidates for being optimal for the auctioneer: The value F^* that induces the incumbent to bid exactly at the entrant's indifference royalty rate $\rho^I(F^*) = r_I^*(F^*)$, where entry occurs (we

assume that the entrant chooses entry when he is indifferent) but revenue is not at its maximum (this is point (a) in the figure), or the value that induces a bid of $\rho^I(F) = 0$ where there is no entry but the auctioneer's revenue is maximal (this is point (b) in the figure). As we noted above the fixed fee that will induce a bid of zero by the incumbent is $F = p^0 + \frac{m+1}{4c} - I$, for ease of notation we will refer to it as F^0 in what follows. The choice between these two values depends on the auctioneer's preferences, i.e. on the weights α and β .

From the definition of the welfare function we can get:

$$\begin{aligned} W(F^0) &\leq W(F^*) \Leftrightarrow AR(F^0) - AR(F^*) \leq \frac{\beta}{\alpha} \\ &\Leftrightarrow p^0 + \frac{m+1}{4c} - I - [p^0 + \frac{m+1}{4c}(1 - \rho^I(F^*)^2) - I] \leq \frac{\beta}{\alpha} \\ &\Leftrightarrow \frac{m+1}{4c} \rho^I(F^*)^2 \leq \frac{\beta}{\alpha} \end{aligned}$$

We can now state our second result in this setting:

Proposition 2 *If the auctioneer cares enough about entry, in the sense that her preferences satisfy:*

$$\frac{\beta}{\alpha} \geq \frac{m+1}{4c} \rho^I(F^*)^2$$

Then she should use the second-price royalty auction with a fixed-fee element, and set the fixed fee to F^ , so that for the given I the incumbent will find it in her interest to bid exactly the royalty rate that makes the entrant indifferent to entry;*

$$\rho^I(F^*) = r_I^*(F^*)$$

If, on the other hand, the auctioneer does not care enough about entry, so that the complimentary inequality holds, then she should run a second-price cash auction instead, (which is equivalent to running the second-price royalty auction with a fixed fee of $F = p^0 + \frac{(m+1)}{4c} - I$).

Chapter 4

The Second-Price Royalty Auction with a Fixed-Fee Element the case of multiple incumbent's types

The results of the analysis in the previous chapter rely heavily on one very important assumption - we assume that the incumbent's type is fixed, and is commonly known to all participants in the game before they have to make any strategic decisions. This assumption, although it simplifies the analysis greatly, cannot be maintained in a fuller, more realistic, setting. The most problematic aspect of the assumption is the postulate that the auctioneer knows with certainty what the incumbent's type is when she designs the fixed-fee schedule, which in turn simplifies the auctioneer's task by reducing it to the choice of a single number.

We now change the basic setting of the model to a more realistic one that includes the possibility for multiple types for the incumbent. In particular we assume that the auctioneer's knowledge of the incumbent's type is similar to her knowledge of the entrant's type; that is, the auctioneer knows only the distribution of the incumbent's fixed cost, but she does not know its actual value. Both the incumbent and the entrant are still assumed to know the value of I with certainty before the game begins. This alternative assumption allows us to investigate the object of central interest - the auctioneer's task in designing the fixed-fee schedule $F(r)$, and its effect

on entry - in a more complete manner, than is possible in the case where a single parameter captures all the problem's complexity.

In this chapter we aim to answer the following questions:

- How should an auctioneer who cares 'enough' (in the sense of the previous chapter) about entry, design a royalty based telecom auction?
- How does the theory compare with the practice?

4.1 Description of the New Model

In our model with multiple types of incumbents, we maintain most of the previous chapter's structure and assumptions, while concealing the value of I from the auctioneer. In particular, we assume it to be common knowledge that the auctioneer has at her disposal information only regarding the distributions of the entrant's and the incumbent's fixed costs;

$$E \sim U[\underline{E}, \overline{E}]$$

$$I \sim U[\underline{I}, \overline{I}]$$

The auctioneer has to base all her decisions, and in particular the design of the schedule of the fixed component of payment in the auction, on this information alone. Furthermore we assume that the distribution of the incumbent's fixed cost is such that the lower bound is not 'too low', in the sense that no matter what the fixed fee might

be, the entrant would not want to enter the auction. This is not a strong assumption. It essentially means that with a zero fixed fee, i.e. with a ‘pure’ royalty auction, the entrant finds it in his interest to enter even if the incumbent’s cost takes its lowest possible value. As we see it, there is little point in discussing the promotion of entry in a situation where nothing could be done to promote it.

This change in the structure of information does not affect the bidders’ strategies. Moreover, the information in the bidders’ possession remains the same; in particular, we maintain the assumption that the value of the incumbent’s cost I is common knowledge between both bidders at the beginning of the game.

4.1.1 The Market

The telecom market at the third stage operates in the same way as in the previous chapter; the winner of the auction serves as a monopolist whose maximum profit takes the form of equation (1):

$$\pi_{\max} = \Delta \frac{(1-r)^2}{4c} + (1-r)p^0 - G$$

where Δ is distributed according to the Gamma distribution with $\mu = \sigma^2 = (m+1)$, c is the constant unit-cost, r is the effective royalty rate, $G \in \{I, E\}$ is the winner’s fixed cost, and p^0 is a large constant insuring the winner never makes a loss. Optimal

values also remain the same:

$$\begin{aligned} q^* &= \Delta \left(\frac{1-r}{2c} \right)^2; \\ p^* &= \frac{2c}{1-r} + \frac{p^0}{\Delta} \left(\frac{2c}{1-r} \right)^2; \\ R^* &= \Delta \left(\frac{1-r}{2c} \right) + p^0; \end{aligned}$$

4.1.2 The Auction

At the time of the auction both bidders are assumed to know the incumbent's type, while the entrant alone knows his own fixed cost. The auction is a second-price royalty auction with a fixed-fee element equivalent to that described in the previous chapter, and thus both bidders will bid in a similar way. The entrant's and incumbent's bidding functions (respectively) take the forms:

$$\begin{aligned} E + F(\rho^E) &= p^0(1 - \rho^E) + \frac{m+1}{4c}(1 - \rho^E)^2 - \frac{\lambda(m+1)}{32c^2}(1 - \rho^E)^4 \\ I + F(\rho^I) &= p^0(1 - \rho^I) + \frac{m+1}{4c}(1 - \rho^I)^2 \end{aligned}$$

where ρ^E and ρ^I are the entrant's and incumbent's bids respectively.

4.1.3 The Entry Stage

We now take a closer look at the entrant's decision regarding whether or not to enter the auction, in order to determine the best way for the auctioneer to go about designing the full $F(r)$ schedule.

As in the previous chapter, one of the most informative parameters in that respect is the entrant's fixed cost that would, if realised, cause him to bid the same royalty rate as the incumbent: $E_{tie} = I - \frac{\lambda(m+1)(1-\rho^I)^4}{32c^2}$. The entrant will win the auction if and only if his fixed cost is lower than or equal to E_{tie} .

Since we assume the entrant knows the incumbent's fixed cost I , but not his own fixed cost, when he has to make his entry decision, we can analyse his behaviour, for a given I , through the formula for E_{tie} (presented in (4) in the previous chapter). It is clear that E_{tie} is dependent only on the incumbent's fixed cost and bid, which in turn is dependent on I , and on the value of the fixed fee at that single point, but not on the entrant's fixed cost or the fixed fee at any other point. We therefore want to claim that *for entry purposes only*, the entrant is indifferent between every fixed-fee schedule provided that it intersects the incumbent's payoff curve at the same point. Figure 9 shows the incumbent's payoff curve with several possible $F(r)$ functions, in Figure 9a they are presented for $I = 0$, while in Figure 9b I is fixed to a positive amount.

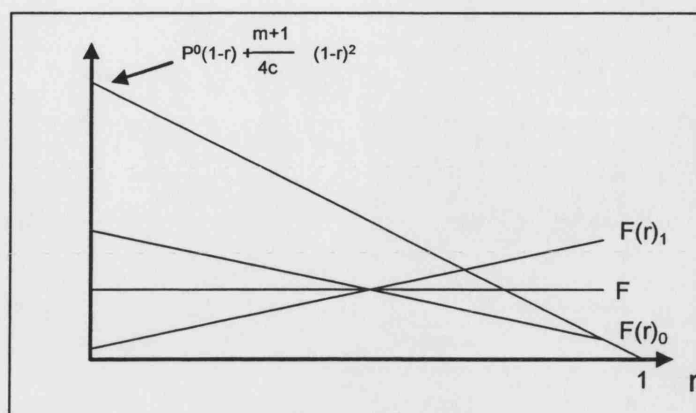


Figure 9a: Several $F(r)$ schedules with $I = 0$.

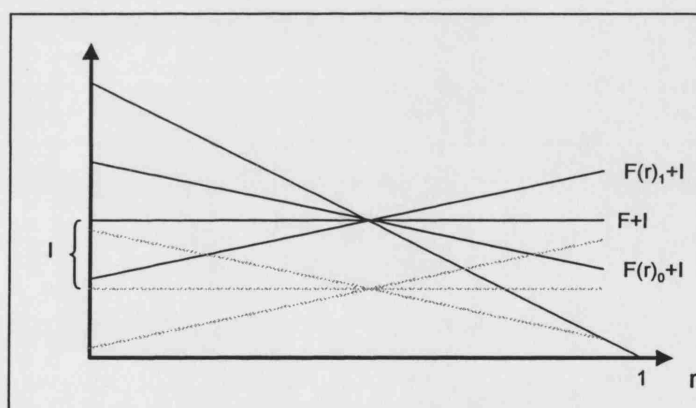


Figure 9b: Several $F(r)$ schedules with $I > 0$.

This means that given that the incumbent's cost is equal to that amount, whichever of the functions in Figure 9 is presented to the bidders at the beginning of the game, as the schedule of the fixed component of the winner's payment, the incumbent's bid and thus the entrant's decision on whether or not to enter would be the same, even

though the entrant's own bid would be different in each case. To see this, note that the value of E_{tie} would be the same, whatever $F(r)$ function is chosen, so long as $F(r) + I$ intersects the incumbent's payoff curve at the same royalty rate $r = \rho^I$. Every possible $F(r)$ creates a one-to-one relationship between the incumbent's bid ρ^I and her fixed cost I , and thus as long as this relationship is unchanged, i.e. as long as I remains the same, the entrant's ex ante position is also unchanged.

In Figure 10 we add the entrant's payoff curve and compare his entry behaviour with a different group of $F(r)$ functions, for a given value of I . Note that I is constant in the figure, the only thing that changes is the fixed-fee schedule $F(r)$.

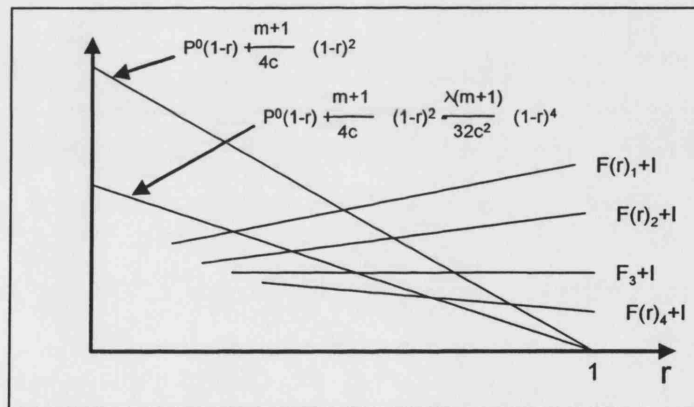


Figure 10: Entry with different $F(r)$ functions.

Which of the above fixed-fee schedules puts the entrant in a more favourable position at the beginning of the auction? The answer is that the entrant's relative position is better the further to the right is the intersection between the fixed-fee schedule and the incumbent's payoff curve. This is the graphical representation of

the positive dependence between E_{tie} and the incumbent's bid, for a given I , that was mentioned in section 3.2.3 and graphically presented in Figure 3.

It might seem counter-intuitive that the entrant is *better off* when the incumbent bids a *higher* royalty rate, but this is due to the fact that, before he learns his own type, the entrant must base his decisions on his expected position relative to that of the incumbent (which he knows), which reflects his chances of winning the auction. The graphical measure of the entrant's relative position in the auction is the vertical distance between the incumbent's and his own payoff curves at the point of the incumbent's bid. This is the amount by which the entrant's fixed cost would have to be smaller than the incumbent's in order for the entrant to win the auction. As is clear from Figure 11 this distance is shorter the greater is the incumbent's bid, and thus the chances of entry occurring would be greatest if the auctioneer would choose $F(r)_4$.

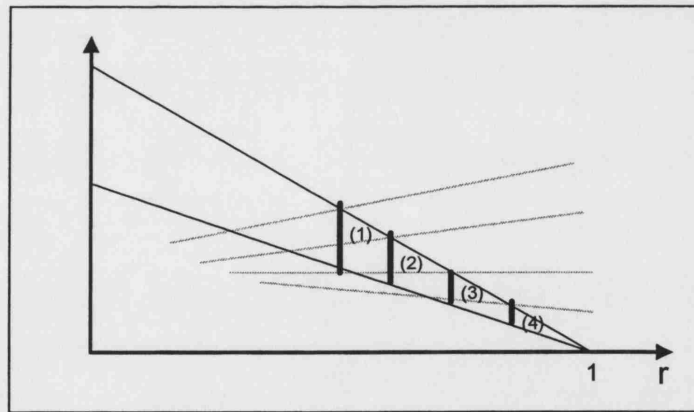


Figure 11: The entrant's disadvantage with different $F(r)$ s.

4.2 The Auctioneer's Task

In the previous section, we described the three stages of the game, where bidders behave exactly the same as in the previous chapter. It is important, however, to understand the exact nature of the entrant's decision on entry, in order to formulate the problem facing the auctioneer in promoting entry via the design of the schedule of the fixed component of payment in the auction. In the last subsection we established two important features of the entrant's entry problem.

Firstly, we saw that *for entry purposes* there is only one point on the graph of the F function that matters - the point of intersection with the incumbent's payoff curve. This is important for the auctioneer since it means she can construct the whole of $F(r)$ point-by-point, dealing with each possible value of I in turn, while keeping track in the process of points of intersection only. This is equivalent to dealing with a constant F for each possible value of I .

Secondly we noted that the chances of entry are increased the further to the right this intersection with the incumbent's payoff curve occurs. This clarifies what the auctioneer should be doing in order to promote entry - for every possible value of I , keep the fixed fee as low as possible, and thus make bids as high as possible. What remains unclear is the way such actions will affect the overall objective of the auctioneer, as entry is not the only aspect of the auction's success we are interested in.

We therefore need to turn back to the auctioneer's preferences in order to establish an objective to be followed in the process of designing the fixed-fee schedule. We continue to assume, as in the previous chapter, that the auctioneer cares both about entry and revenue, as we believe these capture the essence of the telecom auctioneer's environment. We saw in the case of a single type of incumbent in chapter 3, that a trade-off exists between increasing revenue and the chances of entry. This trade-off was very straightforward there due to the assumption that the auctioneer knows I , and can thus calculate the incumbent's bid. When this assumption is relaxed, the situation becomes more complex, and we need to make sure that the nature of this trade-off remains the same.

4.2.1 The Auctioneer's Revenue

All the auctioneer's revenue from the second-price royalty auction with a fixed-fee element must come from one of its two payment components: the fixed fee, paid to the auctioneer directly after the auction is over, and the royalty payment paid from the winner's revenue after one period of operating in the market. We already noted that, when everything else is kept constant, an increase in the fixed fee will cause a drop in the royalty rate, whoever turns out to be the winner. In this sense the auctioneer faces a trade-off between these two sources of income. Therefore, when we come to examine her preferences regarding which of these sources she wants to exploit more, we find that it is helpful to exclude the p^0 term from the formula

of the auctioneer's revenue, as it distorts the balance by over-weighting the royalty component, and making it seem much more profitable than it really is. Recall that the p^0 element was introduced to the market demand in order to keep the algebra of the problem tractable, by not allowing bankruptcies to occur, while not changing the qualitative nature of the problem.

At its simplest, the auctioneer's revenue therefore takes the form:

$$AR(r) = F(r) + r\left[\frac{m+1}{2c}(1-r)\right] \quad (9)$$

where r is the effective royalty rate. The first term in $AR(r)$ is the fixed fee, as determined by the schedule designed by the auctioneer, and the second term is the royalty payment; the effective royalty rate applied to the winner's revenue in the market. The second term has a quadratic form; increasing for $r \in [0, \frac{1}{2}]$, and decreasing for $r \in [\frac{1}{2}, 1]$.

It is clear that the exact shape of the auctioneer's revenue, in the relevant range, is dependent greatly on the shape of $F(r)$. The *optimal* fixed-fee schedule itself however, is dependent on the shape of the auctioneer's revenue, since revenue is one of the two objectives the auctioneer is pursuing in the design of $F(r)$. We shall therefore need to make a couple of alternative assumptions regarding the auctioneer's revenue at this point, then construct the optimal $F(r)$ function on the basis of each of these assumptions, and finally go back and check whether this could be a result of an equilibrium play by all players.

Assumption (1): The Auctioneer's Revenue is Increasing (at least for some of the range; $r \in [\frac{1}{n}, 1]$)

Under this assumption we consider all shapes of $AR(r)$ that are increasing for the larger values of r , including all increasing functions, as well as U-shaped ones (although a U shape is not really realistic for the auctioneer's revenue).

The revenue from the second-price royalty auction with a fixed-fee element can only be increasing with the effective royalty rate if $F(r)$ is itself increasing at least for $r \in [\frac{1}{2}, 1]$, and quite steeply so. Could that be the result of optimally designing the fixed-fee schedule?

If the overall revenue is increasing with r the auctioneer would find it in her interest, both for entry and for revenue considerations, to push the effective royalty rate as high as possible. When both bidders are presented with the $F(r)$ at the beginning of the game, they both can deduce where the point of intersection with the incumbent's payoff curve will occur, as both are assumed to know what I is. This point of intersection determines the incumbent's bid in the auction ρ^I , and whether or not the entrant will choose to enter. See Figure 12a for a generic $F(r)$ schedule that satisfies this assumption.

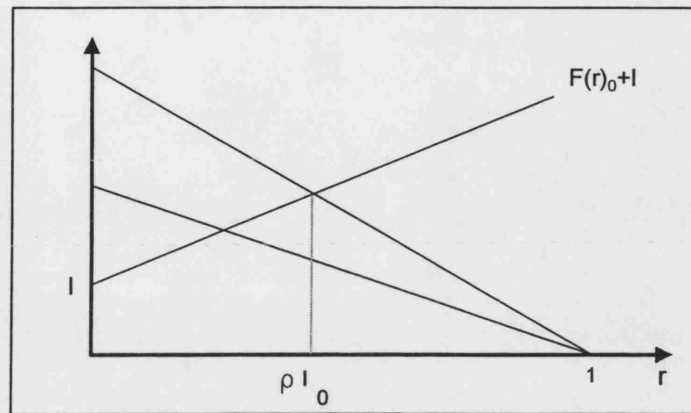


Figure 12a: The incumbent's bid for given I and F .

Two cases exist: (a) for the given I the bidders find that:

$$\rho^I \leq r_I^*$$

in which case the entrant will choose to enter the auction.

And (b) for the given I we have:

$$\rho^I > r_I^*$$

where there will be no entry.

Note that the auctioneer, who does not know what I is, cannot predict which of the two will occur, and thus has actually no, or very little influence on the entrant's decision on entry, (which is the reason for using royalties in the first place.)

More importantly, in both cases the auctioneer has an incentive to push the incumbent's bid further up; if we are in case (a) and the entrant chooses to enter the auction, he will definitely want to enter if ρ^I would have been greater. Since we

assume first that $AR(r)$ is increasing with the royalty rate, revenue will increase as a result as well. If, on the other hand, we are in case (b) and the entrant chooses not to enter the auction, pushing the incumbent's bid up by enough could cause him to change his mind and enter, while increasing the auctioneer's revenue as well.

However, pushing the incumbent's bid up, for any given I , could only be achieved through a reduction in the fixed fee. Applying the same argument over and over again will lead eventually to a 'pure' royalty auction with $F(r) = 0$. See Figure 12b.

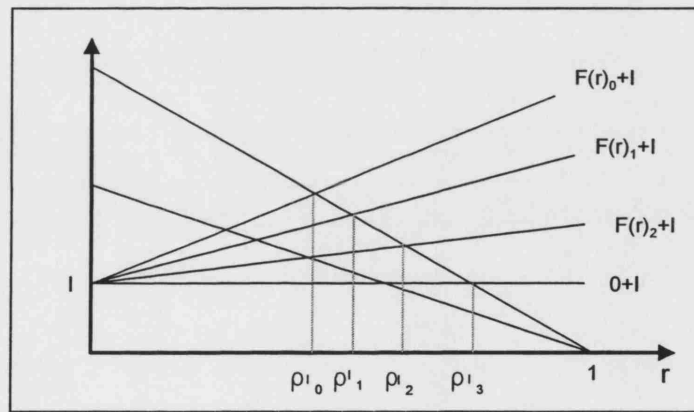


Figure 12b: The incumbent's bid for different $F(r)$ s.

This means that our basic assumption, regarding $AR(r)$ increasing on the range $r \in [\frac{1}{n}, 1]$, is contradicted, (as with $F = 0$ we then return to the quadratic form in the second term of equation (9), which has an *inverted* U shape.)

In equilibrium the auctioneer's revenue from the second-price royalty auction with a fixed-fee element cannot be increasing on the range of possible effective royalty rates $r \in [\frac{1}{n}, 1]$ ¹².

Assumption (2) The Auctioneer's Revenue is Decreasing (at least for some of the range $r \in [\frac{1}{n}, 1]$)

Under this assumption we include all non-increasing, monotonously decreasing, and inverted U shapes for $AR(r)$.

The assumption that the auctioneer's revenue is decreasing with larger values of r is compatible with many different designs for the fixed-fee schedule, one of which is the constant $F(r) = 0$, (under which we would have $n = 2$), but could it be a result of an equilibrium play?

For royalty rates associated with the *decreasing segment* of $AR(r)$, there exists a trade-off between entry and revenue, with higher royalty rates more promotional for entry, and lower ones favourable for increasing revenue. It is no longer clear how an auctioneer who cares both about entry and revenue should go about designing the optimal $F(r)$. Therefore, in order to decide whether an optimally designed fixed-fee schedule could be compatible with the decreasing (or an inverted U shape) auctioneer's revenue curve, we need to be more explicit about the auctioneer's preferences towards entry and revenue.

¹² If we were to leave the p^0 term in the formula for the auctioneer's revenue, then it would in fact be increasing with the effective royalty rate. A 'pure' royalty auction with $F = 0$ would then be optimal.

4.2.2 Entry Takes Priority Over Revenue

As was discussed at length in the first chapter, the telecom auction environment is unique in the central role entry takes in it. The high public profile of telecom auctions makes the auctioneers very aware of the perceived success of the auction. Entry could, and indeed in some cases it was, a major factor in an auction being perceived as a success. In the previous chapter it was possible to construct a welfare function, which weights entry and revenue against each other, for the auctioneer to maximise. This allowed us to identify the group of auctioneers who should use our royalty auction in order to promote entry, according to their preferences, i.e. the exact weights they put on entry and revenue. We called such auctioneers those who care ‘enough’ about entry. With the multiple incumbent type, we want to concentrate on these auctioneers alone - we assume the auctioneer cares enough about entry - and, in particular, we make entry take lexicographic priority over revenue. We believe this reflects the preferences of some telecom auctioneers in quite a close manner, and it allows us to investigate the optimal design of the fixed-fee schedule in a setting where its use is justified. The auctioneers task therefore can be clearly stated as: designing the fixed-fee schedule that would *maximise revenue subject to entry occurring*.

4.2.3 Constructing the Optimal $F(r)$ Point-By-Point

We are now in a position to turn to the actual design of the optimal fixed-fee schedule in the setting described above. Before we describe the algorithm the auctioneer

should use in designing the optimal $F(r)$, we shall restate the set of assumptions under which she is operating.

The Theoretical Environment

- *No cost is 'too low'.* With no fixed fee, i.e. with a 'pure' royalty auction, it is worthwhile for the entrant to enter even with the lowest possible incumbent's cost.
- *Multiple incumbents' types.* The auctioneer has knowledge only of the distributions from which the bidders' valuations are drawn.
- *Entry takes priority over revenue.* The auctioneer wishes to maximise revenue subject to entry occurring.
- *The auctioneer's revenue has an inverted U shape.* The auctioneer's revenue, as a function of the effective royalty rate, is decreasing, at least for the larger values on the relevant range; $r \in [\frac{1}{n}, 1]$ for some n . We will go back to verify this assumption in light of the optimal $F(r)$, to check whether it can be the result of an equilibrium play.

An Algorithm for Designing the Optimal Fixed-Fee Schedule

When going about the design of the optimal $F(r)$ point-by-point, under the assumptions stated above, the auctioneer should follow these steps:

1. Set the fixed fee to $F = 0$.
2. Starting from the lowest, for every possible value of the incumbent's cost I , write down what the incumbent's bid ρ^I would be if that turns out to be her true cost, (and with $F = 0$).
3. Starting from the lowest, for every possible value of the incumbent's cost I , write down what the threshold value for entry r_I^* would be if that turns out to be the true incumbent's cost, (and with $F = 0$).

The auctioneer will find that for every possible value of I , when the fixed fee is set to $F = 0$, we have: $r_I^* \geq \rho^I$. This is due to the (first) assumption of no cost being 'too low' so that the entrant could never gain from entering. The entrant wanting to participate in the auction, for a given set (I, F) , means that when I is the incumbent's cost and F is the fixed fee, he is willing to pay more than the incumbent's bid in royalties before it becomes unprofitable to enter. This is equivalent to having $r_I^* \geq \rho^I$, as r_I^* is by definition the value of the incumbent's bid that makes the entrant exactly indifferent to entry.

4. Starting from the lowest, for every possible value of the incumbent's cost I , gradually increase the constant fixed fee from $F = 0$ upwards until the point where $r_I^* = \rho^I$. When this point is reached write down the corresponding triple (I, F, r) .

By gradually increasing the fixed fee payable by the winner of the auction from $F = 0$ the auctioneer makes both bidders worse off, but the entrant's relative position also worsens. The risk-averse entrant reduces his bid faster than the risk-neutral incumbent as a result of an increase in the fixed fee. It is clear that the entrant, who is willing to enter with $F = 0$, must reach a point where the fixed fee is so high that it is no longer worth his while to enter. That point is where we have $r_I^* = \rho^I$.

5. From the list of triples the auctioneer now holds, one for every possible value of I , ordered from lowest to highest, she should now take the last two entries of each triple; (F, r) and construct the optimal $F(r)$ function.

Every point on the optimal $F(r)$ that results from following the algorithm described above corresponds to a different value of I , and ensures that if this value turns out to be the true incumbent's type, the fixed-fee schedule will induce her to bid such that the entrant will be just indifferent to entry (and thus is assumed to enter). In other words, every point on the optimal $F(r)$ maximises the auctioneer's revenue (under the assumption that it is decreasing) by pushing bids as low as possible, and the fixed fee as high as possible, subject to the entrant choosing to enter, for one possible value of I .

4.2.4 Are We On The Equilibrium Path?

We now need to go back and check whether a fixed-fee schedule that results from the algorithm described above is in fact optimal when all players of the game are interacting non-cooperatively. To this end, it is necessary first to obtain a more precise description of the function we just constructed point-by-point.

What Do We Know About the Optimal $F(r)$?

The optimal $F(r)$ is downwards sloping. To see this, assume that a finite number of values of I are ordered from lowest to highest such that we have:

$$I_0 < I_1 < \cdots < I_N$$

Now, consider first the case of $F = 0$, and note that the smaller I is the better off is the incumbent, or the worse off is the entrant. So, by moving, say, from I_0 to I_1 , and still holding $F = 0$, the entrant's relative position is improved, since, while the incumbent is worse off due to a higher cost, the entrant's chances of winning are increased, while the royalties he would have to pay if he wins drops. This improvement in the entrant's *relative* position in the auction is represented by an increase in the indifference value of entry. So that with $F = 0$ we have:

$$r_{I_0}^* < r_{I_1}^* \cdots < r_{I_N}^*$$

Recall also that we already noted that with $F = 0$ the entrant wants to enter even with the lowest incumbent's cost I_0 , so that we have:

$$\rho^{I_0} < r_{I_0}^*$$

Putting the last two observations together we can conclude that, with $F = 0$ we must have:

$$\rho^{I_N} < \dots < \rho^{I_1} < \rho^{I_0} < r_{I_0}^* < r_{I_1}^* < \dots < r_{I_N}^*$$

In other words, when the fixed fee is constant on $F = 0$, the difference between the incumbent's bid, and the indifference value of it, is increasing with I :

$$(r_{I_0}^* - \rho^{I_0}) < (r_{I_1}^* - \rho^{I_1}) < \dots < (r_{I_N}^* - \rho^{I_N})$$

In the process of designing the optimal fixed-fee schedule the auctioneer gradually increases the constant fixed fee from $F = 0$, starting from the lowest, in turn for every possible value of I . She continues to increase F until the incumbent's bid and the threshold value coincide. Clearly the larger is the 'gap' that needs to be 'bridged' the further will the fixed fee need to be increased in order to close it. This means that on the optimal $F(r)$ the fixed fee will be larger for greater values of cost I , or for smaller values of incumbent's bid ρ^I , (which on the optimal $F(r)$ are the same as the threshold value r_I^*), that is, the optimal $F(r)$ is *downwards slopping*.

The slope of the optimal $F(r)$ is larger (or less negative) than that of the incumbent's payoff curve. To see that it is helpful to look at the graphical representation in Figure 13.

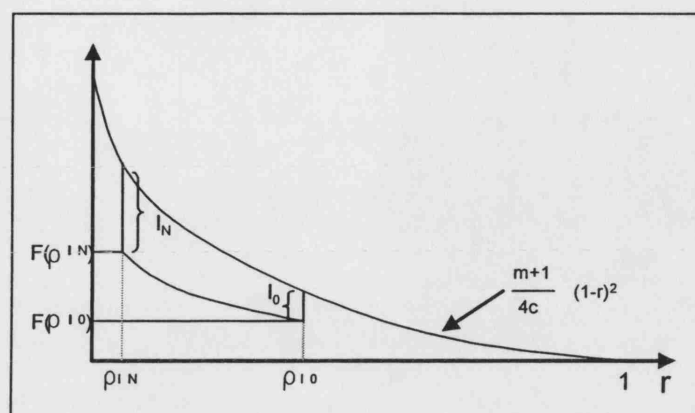


Figure 13a: Deriving the optimal $F(r)$.

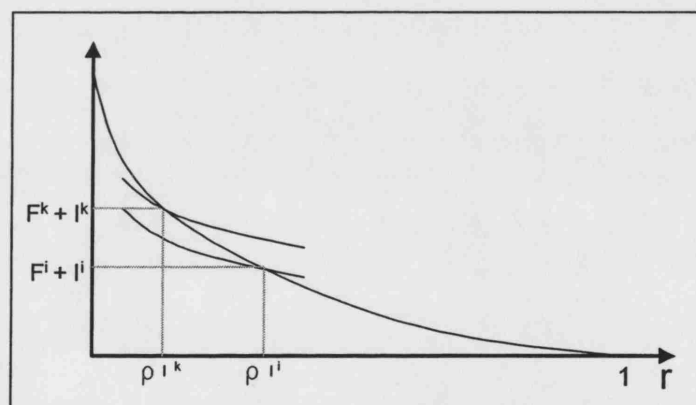


Figure 13b: The optimal $F(r)$ is unique.

Note that the optimal $F(r)$ is designed in such a way that every point on it is an amount of payable fixed fee F that has a corresponding value of cost I which when added to that fixed fee is equated exactly to the incumbent's payoff for the same royalty rate r , so to induce her to make a bid at that point. This means that in order to

get to the shape of the optimal $F(r)$, we can take the incumbent's payoff curve and subtract from every point on it the relevant level of cost.

The optimal $F(r)$ is unique. Note that, by construction, the optimal $F(r)$ induces the incumbent to bid exactly the entrant's indifference value, for every level of I . This means that the argument r of the optimal fixed fee, represents the incumbent's bid *which is equal to the entrant's indifference value*, and not necessarily the effective royalty rate after the auction, (this would also be the effective royalty rate only if the entrant wins the auction). We claim that for every such r there exists a unique pair (I, F) of cost and fee that will equate the incumbent's bid to the entrant's indifference value at exactly that level. To see this look at Figure 13b and note that in order for any royalty rate, say r^i , to become a bid of the incumbent (whether or not it is equal to the entrant's indifference value), the pair (I, F) of cost and fee must sum up exactly to the incumbent's payoff at that royalty rate, call one such pair (I^i, F^i) . This means that for a different such pair, say (I^k, F^k) , to induce the same bid, it too would have to sum up to the same amount. So we would have to have either: $I^i < I^k$ and $F^i > F^k$, or the opposite would have to hold: $I^i > I^k$ and $F^i < F^k$.

Now we go back to use the fact that on the optimal $F(r)$ the incumbent's bid coincides with the entrant's indifference value, and check whether the second pair, (I^k, F^k) , could induce the same incumbent's bid as the pair (I^i, F^i) , *given that this bid is also the entrant's indifference value.*

Consider first case (i) of $I^i < I^k$ and $F^i > F^k$, where the incumbent's cost is increased and the fixed fee is decreased. The increase in I causes the incumbent to reduce her bid, and the entrant's indifference value to increase, while the drop in F causes the incumbent to increase her bid and the entrant's indifference value is increased as well. Since we know that both changes must be equal in magnitude, that is we must have $(I^i + F^i) = (I^k + F^k)$, we can conclude that in case (i), the pair (I^k, F^k) cannot induce the same incumbent's bid as (I^i, F^i) , *given that this bid has to be equal to the entrant's indifference value*. Moreover we can say that in case (i), the switch from (I^i, F^i) to (I^k, F^k) would result in: $r_{I^k}^* > \rho^{I^k}$.

Now consider case (ii) of $I^i > I^k$ and $F^i < F^k$, where the incumbent's cost drops and the fixed fee rises. The drop in I causes the incumbent to increase her bid, while the entrant's indifference value drops as a result. The increase in F causes a drop in the incumbent's bid, and another drop in the entrant's indifference value. A reasoning similar to that in case (i) leads us to the conclusion that in case (ii) too, the pair (I^k, F^k) cannot induce the same incumbent's bid as (I^i, F^i) , *given that this bid has to be equal to the entrant's indifference value*. This time however the switch from (I^i, F^i) to (I^k, F^k) would result in: $r_{I^k}^* < \rho^{I^k}$.

Summing up the two alternative cases above, we reach the uniqueness result:

Proposition 3 *For every possible value for the incumbent's cost I , there exists a single value of fixed fee F that will induce the incumbent's bid ρ^I , and the entrant's indifference value r_I^* , to coincide. The collection of all these F s, one for every possi-*

ble I , and the royalty rates r that correspond to them, is the unique fixed-fee schedule $F(r)$ that has the quality of keeping the entrant indifferent to entry at every point.

From all the features of it that are presented in the last section it is now possible to graphically display and formulate the optimal $F(r)$ in Figure 14.

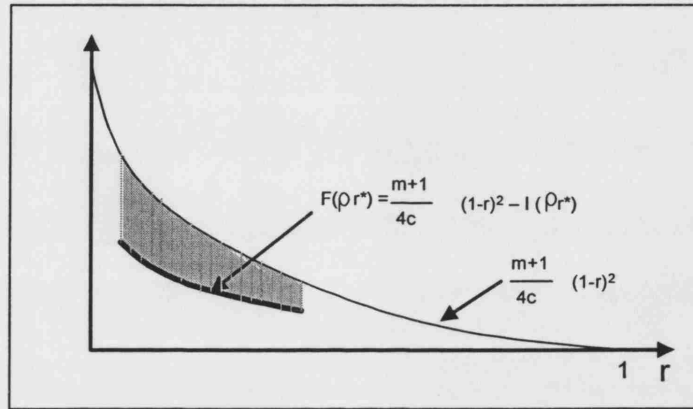


Figure 14: The optimal fixed-fee schedule.

$$F(\rho r^*) = \frac{m+1}{4c} (1 - \rho r^*)^2 - I(\rho r^*)$$

The formula for the optimal $F(r)$ is written with the variable ρr^* as an argument in order to emphasise that along the fixed-fee schedule the incumbent's bid and the entrant's indifference value are united. We already noted in the previous chapter that this united rate does not 'behave' in an equivalent manner to either of its components (ρ^I or r_I^*). In what follows we will therefore refer to the optimal fixed-fee schedule as $F(\rho r^*)$. The last term in the formula for the optimal fixed-fee schedule

$(I(\rho r^*))$ will be referred to as the incumbent's 'cost function' in what follows. It is a decreasing function of its argument - the incumbent's bid *which is equal* to the entrant's indifference value - and it is equal to the shaded area in Figure 14.

Back to the Auctioneer's Revenue

By construction we know that the entrant is kept indifferent to entry (and therefore enters) on every point on the optimal $F(\rho r^*)$, we now need to examine the other component of the auctioneer's objective - her revenue. What shape does it take, and is it actually maximised subject to entry, given the optimal $F(\rho r^*)$?

The simplest form of the auctioneer's revenue is:

$$AR(\rho r^*) = F(\rho r^*) + \rho r^* \left[\frac{m+1}{2c} (1 - \rho r^*) \right]$$

Substituting for the optimal $F(\rho r^*)$:

$$\begin{aligned} AR(\rho r^*) &= \frac{m+1}{4c} (1 - \rho r^*)^2 - I(\rho r^*) + \frac{m+1}{2c} (\rho r^* - (\rho r^*)^2) \\ AR(\rho r^*) &= \frac{m+1}{4c} - \frac{m+1}{4c} (\rho r^*)^2 - I(\rho r^*) \\ AR(\rho r^*) &= \frac{m+1}{4c} (1 - (\rho r^*)^2) - I(\rho r^*) \end{aligned} \tag{10}$$

It is clear from (10) that the precise shape of the auctioneer's revenue is dependent on the shape of its last argument $I(\rho r^*)$ - the incumbent's cost function. Recall however, that the incumbent's cost function must be decreasing with ρr^* , as the larger is the incumbent's cost the better off is the entrant ex ante, or the larger is the *initial* indifference value r_I^* - with $F = 0$. The incumbent's bid, on the other hand is smaller the greater is her cost, and thus the auctioneer will have to increase F further in or-

der to close this 'gap' the larger is I . The two will eventually coincide at a royalty rate smaller than the initial incumbent's bid $\rho^I(F = 0)$.

We can therefore conclude that the auctioneer's revenue is indeed decreasing given the optimal fixed-fee schedule, and thus she would not have an incentive to deviate from this design which keeps bids as low as possible without hurting chances of entry.

Do Bidders Have an Incentive to Deviate?

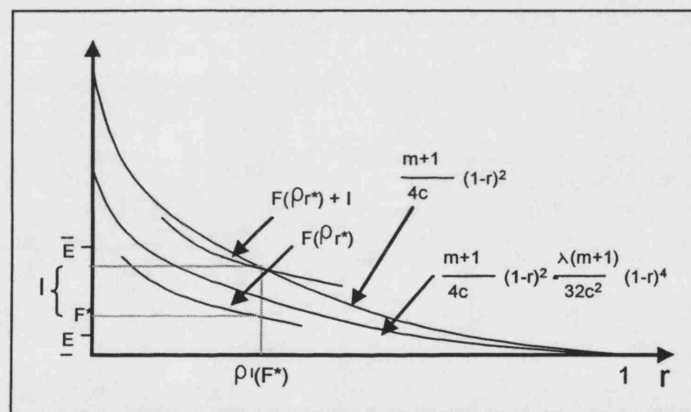


Figure 15: Bidders' initial information.

When bidders are presented with the optimal fixed-fee schedule at the beginning of the game, both are assumed to know I and the distribution from which E is drawn (the entrant alone will learn the value of E if he chooses to enter the auction). In Figure 15 all the information at the bidders disposal is presented.

The entrant has two decisions to make, the first is whether or not to enter the auction, and the second is choosing a bid, after he learns his cost upon entering. Recall that, as was discussed at length in the first chapter, the concept of entry that the auctioneer is trying to promote via the design of the second-price royalty auction with a fixed-fee element is captured in this setting by the entrant deciding to enter and participate in the auction, rather than by his winning it. We therefore need to examine whether an entrant who is presented with the optimal fixed-fee schedule at the beginning of the game has an incentive to deviate from deciding to enter, rather than what his bid will be if he does enter. By construction the optimal $F(\rho r^*)$ keeps the entrant indifferent to entry for every possible value of I and so at the time he has to make his entry decision, before he learns his own cost, there is no reason for the entrant to deviate from entering.

What happens in the auction itself with the optimal $F(\rho r^*)$?

After he chooses to enter the entrant learns his cost and the auction is about to begin. The incumbent places her bid ρ^I at the point where $F(\rho r^*)$ intersects her payoff curve (minus her true cost). We already saw that this is the only bid the incumbent wants to make; all smaller royalty rates could be profitably increased, while any higher rate will, on average, cause a loss. This is due to the fact, mentioned above, that $F(\rho r^*)$ has a less negative slope than the incumbent's payoff curve. In order to determine the entrant's bidding behaviour we need to verify that his payoff curve is also steeper, i.e. has a more negative slope, than $F(\cdot)$.

The entrant's payoff curve is:

$$\frac{m+1}{4c}(1-r)^2 - \frac{\lambda(m+1)}{32c^2}(1-r)^4$$

Its slope is:

$$\frac{m+1}{2c}r - \frac{m+1}{2c} + \frac{\lambda(m+1)}{8c^2}(1-r)^3$$

For comparison the slope of the optimal $F(\rho r^*)$ is:

$$\frac{m+1}{2c}\rho r^* - \frac{m+1}{2c} - \frac{dI(\rho r^*)}{d\rho r^*}$$

It is clear that the third term in both expressions is the one that will determine which is greater. Recall that the incumbent's cost function $I(\rho r^*)$ gives the incumbent's cost for every value of the incumbent's bid, *given that it is equal to the entrant's indifference value*. We know that this function is decreasing, but the rate at which it decreases is dependent on the slopes of both the incumbent's and the entrant's payoff curves.

To see this it is useful to look at the graphic representation in Figure 16.

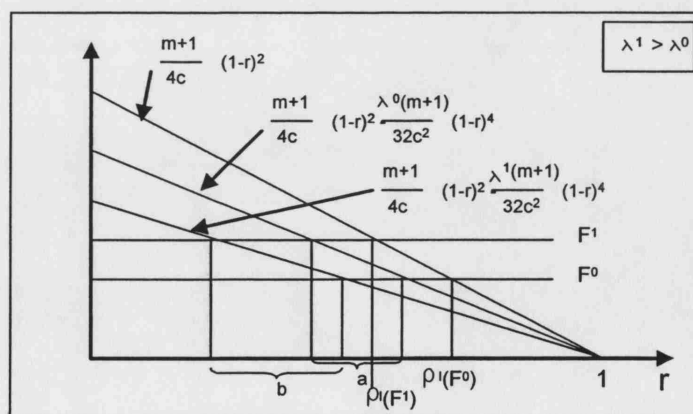


Figure 16: The optimal $F(r)$ for different λ values.

In the figure there is a sketch of an incumbent's payoff curve with two different payoff curves for the entrant, one steeper than the other. Now recall that the process of designing the optimal fixed-fee schedule involves increasing the fixed fee gradually from zero, until the incumbent's bid is united with the entrant's indifference value. Both values will decrease as a result of the increase in F , but the indifference value $r_I^*(F)$ will decrease faster the flatter is the entrant's payoff curve. This is due to the fact that the flatter is the entrant's payoff curve (relative to that of the incumbent which serves as a benchmark), i.e. the more risk-averse the entrant is (relative to the risk-neutral incumbent), the faster he reduces his own bid as a result of an increase in the fixed fee (see the difference between a and b in Figure 16), and thus the faster does his *relative* position in the auction worsen, and the faster his indifference value $r_I^*(F)$ drops.

What happens to the optimal fixed-fee schedule as a result? If the entrant's indifference value decreases faster relative to the incumbent's bid then the two can be united faster, i.e. with a smaller increase in F and at a higher royalty rate.

The corresponding optimal $F(\rho r^*)$ is therefore flatter, the flatter is the entrant's payoff curve, or the more risk-averse he is. This still does not insure that the optimal fixed-fee schedule is indeed flatter than the entrant's indifferent curve, which is what we need for him to stick to his bidding strategy and not deviate. In order to insure this we need to make an assumption about the boundaries of the incumbent's cost I_0

and I_N (or \underline{I} and \bar{I}). The condition we need satisfied is:

$$\begin{aligned}
 I_N - I_0 &\geq \frac{\lambda(m+1)}{32c^2}(1 - \rho r_{I_N}^*) - \frac{\lambda(m+1)}{32c^2}(1 - \rho r_{I_0}^*) \\
 I_N - I_0 &\geq \frac{\lambda(m+1)}{32c^2}(\rho r_{I_0}^* - \rho r_{I_N}^*) \\
 \frac{I_N - I_0}{\rho r_{I_0}^* - \rho r_{I_N}^*} &\geq \frac{\lambda(m+1)}{32c^2}
 \end{aligned} \tag{11}$$

The meaning of the condition in (11) is that the absolute value of the average slope of the incumbent's cost function is greater than or equal to an amount that could be thought of as a measure of the entrant's disadvantage due to his risk-aversion. We know that the incumbent's cost function is decreasing, so if the absolute value of its slope is greater it means the function is decreasing more steeply. This condition is more likely to be satisfied, for a given degree of risk-aversion of the entrant, the larger is the upper boundary on the incumbent's cost I_N . To see this condition graphically see Figure 17.

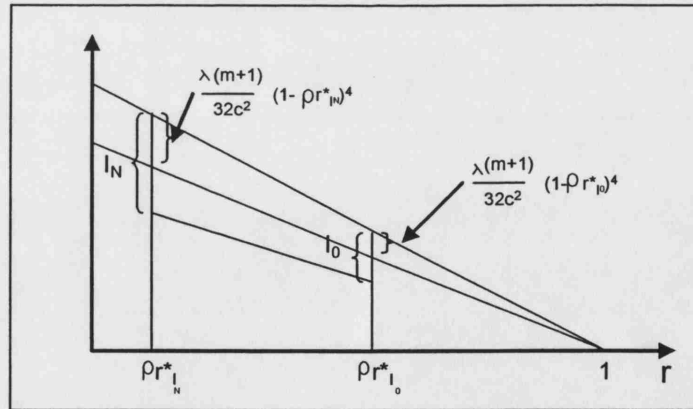


Figure 17: The optimal $F(r)$ for different $(I_N - I_0)$.

The figure shows a generic case for both bidders' payoff functions, and the optimal fixed fee. The optimal $F(\rho r^*)$ will lie somewhere between 'the incumbent's payoff minus I_N ' and 'the incumbent's payoff minus I_0 '. It is clear that, for a given level of risk-aversion for the entrant, which determines where the incumbent's bid is united with the entrant's indifferent value, the optimal $F(\rho r^*)$ is flatter the larger is $(I_N - I_0)$.

We can now conclude that as long as the upper bound on the incumbent's cost I_N is high enough relative to the lower bound, such that the condition in (11) holds, the entrant has no incentive to deviate from his strategy of choosing 'entry' at the first stage (he is assumed to enter when indifferent), and then choose his bid according to his realised cost, at the point of intersection between his payoff curve minus his cost and the optimal fixed-fee schedule.

4.2.5 The Final Shape of the Optimal Fixed-Fee Schedule

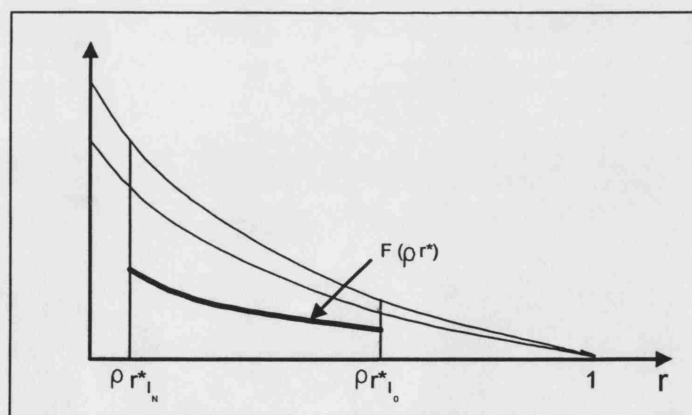


Figure 18: The optimal fixed-fee schedule.

Figure 18 shows a generic case for the shape of the optimal $F(\rho r^*)$. Note that it is not defined on the whole range of possible royalty rates, i.e. on $[0, 1]$, but only on the interval $[\rho r_{I_N}^*, \rho r_{I_0}^*]$, with $0 \leq \rho r_{I_N}^* \leq \rho r_{I_0}^* \leq 1$. These are reasonable restrictions on bids, as the auctioneer is likely to want to put an upper bound on the distortive royalty bids at the point where they do not promote entry anymore. The lowest royalty rate submittable as a bid in the auction - $\rho r_{I_N}^*$ - serves as the reservation royalty rate (r_0) in this case of multiple incumbent's types.

4.3 The Hong Kong Royalty Auction

Now that we have completed the design of the 'second-price royalty auction with a fixed-fee element', it is interesting to compare it with the design of the only real-world auction of the sort. The 3G telecom auction held in Hong Kong in 2001 was, to the best of our knowledge, the only use of a royalty based auction in the telecom industry. The design of the HK auction was, however quite different than that of the second-price royalty auction with a fixed-fee element developed and analysed here. We now take a closer look at the HK auction with its differences and similarities to our model, with the aim of better understanding the situation faced by the HK potential bidders before the auction.

4.3.1 The Basic HK Auction Design

In the HK market of telecommunications services there were 6 (having fallen from an initial 8) operators of 2nd generation technology, prior to the announcement on July 2001 of the government's intention to auction 4 licences for operating 3rd generation technology. Bidders who applied and approved for participation were to bid in the auction in terms of royalty rates $r \in [0, 1]$, the four winners of the licences were the bidders whose bids were highest. The single 'effective royalty rate', in the terms of our model, would then be set equal to the fifth highest rate. At this point it is possible to note one of the important features of the HK auction that is similar to our model - both auctions are essentially second-price auctions, where it is well known that bidders have an incentive to bid their true valuations. The effective royalty rate would be used to calculate the winners' royalty payments, at several points in time, each at the end of a pre-specified period of operating in the market.

The HK auction had a fixed-fee element to it as well. The published rules of the auction included a 'schedule of minimum payments'. This fixed-fee schedule was an *increasing* function of the royalty rate, known to all potential bidders in advance, and it was introduced to ensure that payments would not run below some minimum acceptable level. We now get to the most important difference between the HK auction and our model - the payment rule. In the HK auction the winners were committed to pay the auctioneer *either* a royalty payment of the effective royalty rate applied to their revenues (or 'turnovers' as the rules specified), *or* a fixed fee - cash payment

equal to the amount specified by the fixed-fee schedule at the effective royalty rate - whichever turns out to be greater, but not both. In the notation used throughout we can write the HK payment rule as: $\max\{rR, F(r)\}$. This is a very important difference, which substantially changes the environment in which the bidders operate. Essentially the HK auction is *either* a 'pure' royalty auction, *or* a simple cash auction, but the bidders are unable to say which is the case before the end of the bidding competition. Each of these auction formats present bidders with different incentives as to their bidding behaviour. To analyse this auction design further we need to make some modifications in our model, so that it can serve as a simplified approximation to the HK auction.

4.3.2 A Simple Analysis of the HK Auction Environment

In analysing the HK case we want to focus on the auction itself, and in particular on the effect that the 'dichotomy' in the payment rule has on the bidders' decisions. For this purpose we leave all assumptions made in the first and third stages of the game intact, while we concentrate on the second - auction - stage¹³.

How Should the Winner of the HK Auction Operate in the Market?

We continue to assume that after the auction all variables of the telecom market are revealed, and so the winner, whoever it might be, can act optimally in the production process.

¹³ A full analysis of the HK telecom market lies outside the scope of this study.

Recall however that in the HK design, only after the auction is over the winners are able to determine whether they are going to operate in the market under a ‘royalty regime’ where a royalty payment is to be made to the auctioneer at the end of the licence period, or under a ‘fixed-fee regime’ where a cash payment is to be paid equal to the amount specified in the fixed-fee schedule. Which one of the two regimes will exist after the auction depends on the results of the bidding competition, which determine the effective royalty rate r , and on the revenue the winner makes in the market (together with the fixed-fee schedule). We therefore effectively have two alternative telecom markets environments; one for each possible regime, each yields a different revenue:

$$R_{(roy)} = (1 - r) \frac{m + 1}{2c} + p^0 \quad (12a)$$

$$R_{(fix)} = \frac{m + 1}{2c} + p^0 \quad (12b)$$

These expressions are taken from the optimal market revenue in the market analysis in the third chapter, and are exclusive of the payment to the auctioneer in both cases.

However, the revenue the winner gains depends on the actions he or she takes in the market, which in turn will depend on the payment regime. Note that for all $r \in [0, 1]$, when the winner is operating optimally we have $R_{(fix)} \geq R_{(roy)}$. This means that there could exist a range of values for the effective royalty rate, for which we will find that:

$$r R_{(roy)} \leq F(r) \leq r R_{(fix)}$$

What payment regime should operate in this situation? If the winner expects the royalty payment regime to stand he or she will reveal in the end of the licence period (recall that we assume revenues are observable) a revenue of $R_{(roy)}$, in which case the auctioneer will demand the fixed-fee payment which will be larger, and the winner will have found that his conduct in the market was not optimal. If on the other hand, the winner expects the fixed-fee regime to stand, then at the end of the licence period he or she will reveal a revenue of $R_{(fix)}$, and the auctioneer will demand the royalty payment, applied to that amount, instead. Values of the royalty rate for which the above inequality holds will therefore determine a third regime - 'the intermediate regime'.

We can now state the most problematic feature of the HK auction design - there is no way for the winner of the auction to operate *optimally* in the telecom market when the effective royalty rate turns out to lie in this 'intermediate range' where $rR_{(roy)} \leq F(r) \leq rR_{(fix)}$.

In the intermediate regime the best the winner can do in order to avoid a loss is to operate in the market in an un-optimal manner that will restrict his or her fixed revenue $R_{(fix)}$ to equal exactly to the payable fixed fee in this range, and so to make sure he or she remains in the fixed-fee regime. In other words, if the effective royalty rate turns out to be in the intermediate regime range, the winner of the auction should operate in the market in a way that ensures that $R_{(fix)} = F(r)$, and thus making sure he or she will just break even in this case.

How big is the intermediate range?

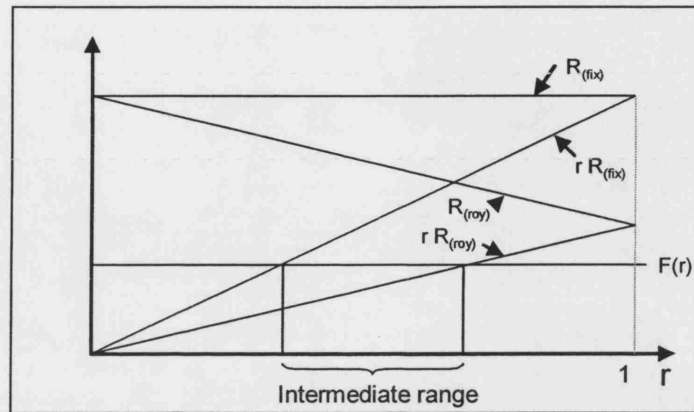


Figure 19: The Hong Kong royalty auction.

Figure 19 sketches the possible values of the optimal revenues and royalty payments depending on the effective royalty rate. It is clear that the size of the intermediate range depends on the shape of the fixed-fee schedule, and could be quite significant, (especially with an upwards sloping $F(r)$).

How Should Bidders Bid in the HK Auction?

We now turn to the analysis of the basic design of the HK auction, *before any bidder-types are introduced*.

Before the auction begins the two bidders are presented with the (upwards sloping) fixed-fee schedule $F(r)$, bidders then place bids in terms of royalty rates $r \in [0, 1]$, and the winner is whoever bids the highest rate. The effective royalty rate is set at the second highest bid, which will then be used to calculate either a roy-

alty payment or a cash payment through the fixed-fee function, whichever turns out to be greater. This means that at the end of the auction the winner faces these three options:

- (a) $F(r) \leq rR_{(roy)}$ In which case the winner operates in a ‘pure’ royalty market for the licence period, at the end of which the royalty payment is due.
- (b) $rR_{(roy)} \leq F(r) \leq rR_{(fix)}$ In which case the winner is in the intermediate regime.
- (c) $F(r) > rR_{(fix)}$ In which case the winner is committed to pay a fee for the licence determined by the fixed-fee schedule.

With no fixed costs introduced, all bidders care about is the revenue they can make by operating optimally in the market, minus the payment they make to the auctioneer. In other words the winner will gain either $(1 - r)R_{(roy)}$ in case (a) or $R_{(fix)} - F(r)$ in case (c) above, in the intermediate case (b) the winner will make a profit of zero.

The incentives that drive bidders in their bidding behaviour depend also on the shape of the fixed-fee schedule, which in turn determines the order in which the three regimes appear on the interval $r \in [0, 1]$. An upwards sloping $F(r)$ (of the form used in HK) may start below the $rR_{(roy)}$ line with the royalty regime thus corresponding to low values of r , then there will be a phase of intermediate regime before the fixed-fee regime will correspond to the higher values of royalty rate. The opposite is likely to occur with a constant or a downward slopping $F(r)$. Figure 20 shows a few examples.

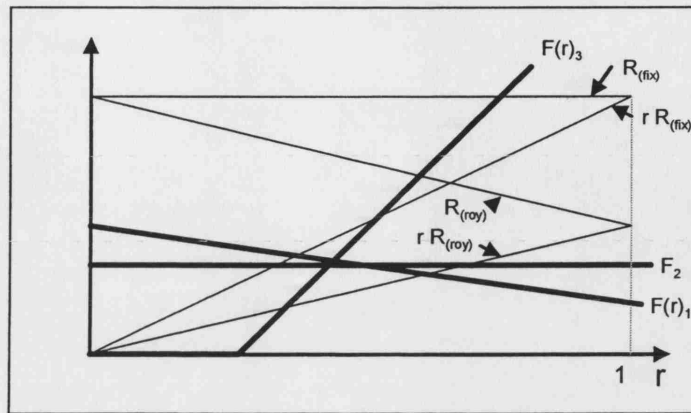


Figure 20: Different fixed-fee schedules for HK.

In the actual HK auction potential bidders were presented with a list of payable fixed fees, each corresponding to a certain effective royalty rate, and to a stage in the licence period. Transferring it to a graphical form gives a fixed-fee schedule of the following form.

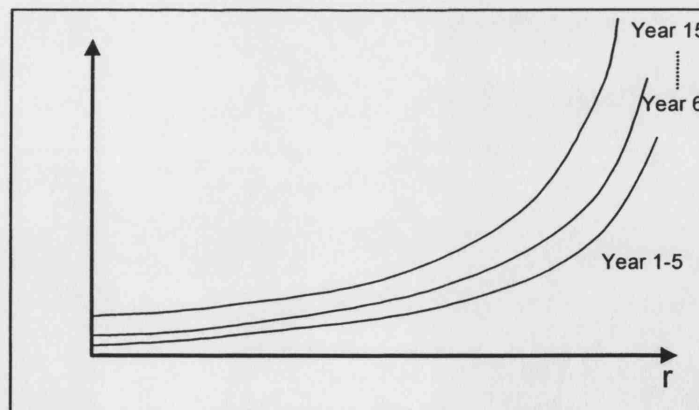
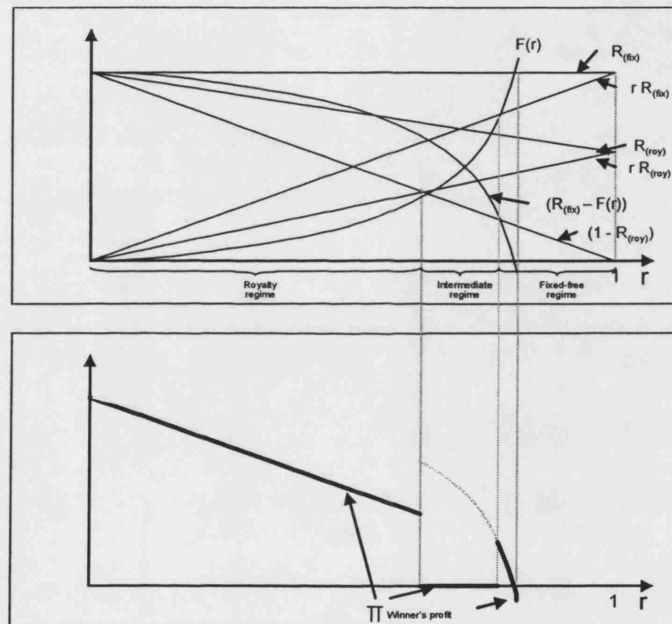


Figure 21: A sketch of the HK fixed-fee schedule.

Since we only have a single period in our model, we will deal with a single-valued $F(r)$ function that capture the essence of the relationship between the fixed fee and the royalty rate that exists in the HK schedule. We can now try to better understand the situation faced by the HK bidders when they consider the best way to bid. In Figure 22a we show a graphic representation of the generic case for a HK bidder, still with no fixed costs, Figure 22b shows the corresponding bidder's profit.



Figures 22a and 22b: Deriving the HK bidder's profit.

We can see that the intermediate regime causes a reduction in the size of the fixed-fee regime, and pushes the winner's profit to zero for that part. The formula for this profit schedule would take the form:

$$\pi = \left\{ \begin{array}{ll} (1-r)R_{(roy)} & \text{for } 0 \leq r \leq r_1 \\ 0 & \text{for } r_1 \leq r \leq r_2 \\ R_{(fix)} - F(r) & \text{for } r_2 \leq r \leq r_3 \end{array} \right\}$$

where r_1 is the rate of royalties for which $rR_{(roy)} = F(r)$, and so the royalty regime ceases to be available, r_2 is the rate where $rR_{(fix)} = F(r)$, and the intermediate regime is replaced by the fixed-fee regime, and r_3 is the rate where $R_{(fix)} = F(r)$, and thus the winner is unable to increase his bid above r_3 without risking a loss. Deriving the precise bidding functions for the HK bidders is not, however, among our aims in this analysis. Instead we want to keep the discussion without any bidder types, and point to some general problems in the HK auction design, that might affect the auction's performance in terms of entry.

The most important feature, entry-wise, in the HK auction design is the fact that it is not necessarily a royalty auction, in the sense that if the effective royalty rate falls in the fixed-fee range, then the auction is effectively a regular cash second-price auction. Moreover, with the HK style dichotomy in the payment rule what the auctioneer is actually saying to the potential bidders is: if conditions in the telecom market turn out to be favourable, then I will take a share of this high revenue, but if on the other hand conditions in the market are revealed as poor, then you would have to pay upon winning a cash amount that has no relevance to your revenue. This arrangement does not offer potential bidders an insurance against bad outcomes, on the contrary it offers such an insurance to the auctioneer.

Another feature to consider is the HK choice of an upwards sloping fixed-fee schedule which, although it seems the intuitive choice, has been shown here to be in fact non-entry-promoting. In the HK setting an upwards sloping $F(r)$ means that the fixed-fee regime becomes available with the larger values of the effective royalty rate, which makes it more likely that at the end of an active bidding competition bids will reach this payment regime. In other words, in order to increase the chances of ending up in the royalty regime after the auction, a bidder may have to restrict his bidding to rates below r_2 which, if this bidder is risk-averse, could help him in the market stage, but it would hurt him in the auction and decrease his chances of winning it. On its own, the fact that the auction is conducted in terms of royalty rates is not sufficient for promoting entry of the more risk averse bidders, if its likely to lead to a large cash payment after the auction.

And lastly, the HK auction was based on the second-price rather than the first-price setting, as the price was determined by the highest losing bid rather than the lowest winning one which, (if everything else is equal), is thought to be a format much more suitable for attracting new comers. This issue of the entry promoting properties of the ascending and descending price auctions is explored in the next chapter.

Our analysis of the HK auction leads us to the conclusion that a risk-averse entrant has no extra incentive to enter this royalty auction rather than the typical second-price cash auction, even though it is conducted in royalty terms.

Chapter 5

Entry Into Auctions - An Experimental Approach¹⁴

In this chapter we abandon the use of royalties in auctions altogether, and turn to an experimental examination of the issue of entry within two of the most common auction formats in the telecom industry - the ascending price auction and the first-price sealed bid auction (the latter is strategically equivalent to the descending price auction, which will actually be used in our experiment). Our aim in this chapter is to compare the entry behaviour of bidders in a monitored environment where the only variable is the auction format. Moreover, we try to isolate the entry properties of these auctions, one of which preserves its efficiency in the presence of asymmetries (the ascending price auction) while the other may result in an inefficient outcome in such circumstances (the first-price auction). It therefore seems sensible to expect, even before dwelling on the theory, that the first-price setting would better promote the entry of weaker bidders in an asymmetric environment.

The question of whether to use a first-price or a second-price setting in telecom auctions (or as is usually more suitable for the telecom market, a more complicated design which is based on one of these two simple mechanisms) had been raised in debates among auctioneers and auction designers in many occasions. To answer this question in a general manner would be impossible, as different auctioneers in

¹⁴ This chapter is based on joint work with Joe Swierzbinski and Chris Tomlinson.

different markets may have different priorities, but one aspect of the answer should involve entry considerations, and the influence that each of these settings has on its chances to occur. Indeed, even this one aspect of the auction design seems to raise grounds for a debate among theorists, as well as a large diversity in evidence from the practical use of these auction formats - while the UK auction was essentially an ascending-price auction and a huge success in terms of entry, the Danish auction was based on the multi-object version of a first-price auction and still managed to attract new comers to the bidding, one of whom even displaced an incumbent.

In their 2001 paper Abbink, Irlenbusch, Pezanis-Christou, Rockenbach, Sadrieh and Selten, conduct an experimental test of alternative designs for the UK auction. One of the results they report deals with the "Chances for new entrants to purchase a licence", where they claim that their experimental evidence contradicts the opinion of the UK auction designers, who suggested that the auction include a sealed-bid phase at the end, in order to attract entry (this design was not used in the actual UK auction, as when an additional licence was offered, entry was thought to be better promoted with a simpler design). Abbink et.al. report that "the highest number of successful new entrants can be found in the English auction" (p.13).

This claim stands in contrast to the views expressed by some of the economists involved in the design of the UK auction (see Binmore and Klemperer 2002, and Klemperer 2002) who claim that "Sealed-bid auctions do better at promoting entry because they give entrants a better chance of winning against strong incumbents"

(Binmore and Klemperer 2002, p.C83). The source of this apparent disagreement probably lies in the different definitions the authors give to the term ‘entry’, which the auction is expected to promote. While Abbink et.al. measure whether the auction format promotes entry-to-the-telecom-market, by newcomers actually purchasing a licence, Binmore and Klemperer claim that newcomers might not bother to participate in the bidding of an ascending-price auction in the first place, and thus obviously would have no chance of winning it. They therefore refer to entry-to-the-auction as the measure needed to be promoted by the auction design, (see Klemperer 2002 for a full exposition of this argument).

This is the property of the two common auction formats that we aim to test in this chapter. The setup of the experiment is constructed so that it can be easily related to a typical telecom environment where asymmetries between bidders’ valuations are usually present (at least *ex ante*) and may affect the weak bidders’ entry behaviour.

The experiment is built on a theoretical analysis included in Vickrey’s seminal paper of 1961, where he analyses and compares the equilibrium bidding strategies of two asymmetric bidders in the first-price, sealed-bid auction, and the second-price, sealed bid auction (or ascending auction). Vickrey notes that in the framework of the first-price auction, a complete analysis of bidding strategies in the presence of asymmetries in valuations creates a very complicated, highly intractable problem. He therefore turns to the analysis of an example, on the basis of which our experiment is built.

5.1 Vickrey's Example

In Vickrey 1961 section II, under the 'Non-homogeneous case' the following example is analysed; Two bidders compete over a single indivisible item. Bidder 1's valuation v_1 is drawn from a uniform distribution on $[0, 1]$, while bidder 2 has a known, fixed, valuation $v_2 = a$.

In an ascending price auction ('common auction' in Vickrey's terms) the (weakly) dominant bidding strategy of both bidders is to bid their true valuation.

In the descending price ('Dutch' or first-price sealed) auction, which is the main focus of the Vickrey example, the equilibrium analysis is much more complex. Vickrey notes that in order for an equilibrium to exist, bidder 2, whose valuation is fixed and known to both bidders, must be using a mixed strategy in which he randomises over some range of bids, strictly below his valuation, while bidder 1 uses a single-valued bidding function. This feature of the equilibrium has interesting entry implications, and is central to the construction of our experiment.

This type of asymmetry between bidders' valuations could put bidder 2 in an advantageous position, if his fixed valuation a is large enough relative to the upper bound of bidder 1's valuation (in Vickrey's example this bound is normalised to 1).

Assume for the moment that a is actually equal to the upper bound of bidder 1's valuation, i.e. that $a = 1$. It is clear that in an ascending price auction, bidder 1 has no chance of winning any positive surplus, and thus, if participating in this auction is costly, bidder 1 will choose not to participate. However, if a descending price auction

is used instead, the same bidder 1, from an ex ante point of view (when he knows the distribution of his valuation but not its realised value) might find it worthwhile entering and participating in the bidding, even if it involves some cost.

A telecom auction, as was discussed in the previous chapters, might give rise to a similar situation, in which there exists at least one bidder whose valuation is known to dominate that of others in some sense. Typically, the strong bidders are the incumbents in the market, while the weaker ones are potential new entrants, whose participation in the auction, or the promotion of that participation, is at the focus of this work. Vickrey's example provides an opportunity to test and compare empirically, in a simple laboratory setting, the features of two of the most commonly used auction formats, in terms of their 'entry friendliness'.

5.2 Experimental Setting

In our experiment there are two types of bidders; A strong bidder - Topcat - whose valuation is fixed and known to both bidders to be set at \$100 million, and a weak bidder - Underdog - whose valuation, in millions of dollars, is known to be drawn from a uniform distribution on the interval $[0, 100]$. The actual realisation of Underdog's valuation is revealed privately to the subject playing the Underdog role. Every time a subject is assigned the Underdog role in a certain auction format (ascending or descending price), his or her valuation is a new draw from the same distribution,

however across the two formats (the two treatments of the experiment) the same set of Underdog valuations is used to allow for comparability.

Every auction in the experiment pairs one of 5 Topcats with one of 5 Underdogs; each pair competes for a single telecom licence, in either an ascending or a descending price auction. Every round in the experiment is constructed of 4 such auctions. Pairings are changed in an unpredictable manner after each auction (in order to minimise the opportunity for collusion), and the players' roles are alternated (so that each subject plays the role of Underdog and the role of Topcat an equal number of times), but the auction format is constant throughout the experiment run.

A single experiment run is constructed of 6 rounds of auctions. In the first 3 rounds (i.e. 12 auctions) participation is free of charge, in rounds 4-6 (12 auctions) bidders who want to participate in the auction are required to pay an entry fee. If a subject decides in this part of the experiment not to pay the entry fee the auction is cancelled. After both bidders make their entry decisions it is announced whether the licence is awarded to the one bidder who paid the entry fee for free (the reservation price is set to zero), or in the case of both bidders choosing not to participate, the licence remains unsold. Only if both bidders pay the entry fee, an auction takes place. It is important to note that both bidders have to make the decision on whether or not to pay and bid in the auction *after* they are informed of the roles they will play in that auction, but *before* Underdog's valuation is drawn and revealed. The fee for

participating in an auction in rounds 4-6 was constant throughout the experiment, and set to \$2.66m, (the way in which the entry fee was determined is analysed below).

Subjects are rewarded in the form of lottery tickets. This is done in an attempt to neutralise any possible variations in risk attitude that subjects might hold, and that could affect the bidding in the descending price auction (in the ascending price auction optimal bidding behaviour is unchanged in the presence of risk-aversion), and induce all subjects to behave as if they were risk-neutral. This method was first analysed by Roth & Malouf (1979), where they identify the effect of separating the chance of winning the auction, which is a positive function of the bid, and the profit gained in the auction if won, which is a positive function of the valuation, from the size of the monetary prize available for the winner, which is constant.

This separation is achieved by converting the winner's profits in the auction into chances of winning a fixed monetary prize, at some known conversion rate. After the auction, a lottery takes place, where the winner can either win the fixed prize or win nothing, in which case he or she is in the same position as the losing bidders in the auction. This way, the incentives of bidders in the auction are preserved, while the probability of winning the monetary prize increases *linearly* with the auction profit. Rational bidders therefore will behave as if they were risk-neutral, whatever their actual risk attitude may be. Formally this argument can be shown as:¹⁵

¹⁵ This demonstration is taken from Cox, Smith & Walker (1985).

The winner of the auction participates in a lottery, in which he or she can win a prize of:

$$\begin{aligned} x & \text{ with probability } \frac{v_i - b_i}{\bar{v}} \\ 0 & \text{ with probability } 1 - \frac{v_i - b_i}{\bar{v}} \end{aligned}$$

where v_i is the winner's valuation, b_i is the winner's bid, (i.e. the highest bid in the auction), and \bar{v} is the maximum possible valuation.

Bidder i's problem is therefore to:

$$\max\{G(b_i)[u_i(x)(\frac{v_i - b_i}{\bar{v}}) + u_i(0)(1 - \frac{v_i - b_i}{\bar{v}})] + (1 - G(b_i))u_i(0)\}$$

where $G(b_i)$ is the probability that i's bid is the highest, and $u_i(\cdot)$ is bidder i's possibly non-linear utility function.

$$\begin{aligned} & \max\{G(b_i)u_i(x)(\frac{v_i - b_i}{\bar{v}}) + G(b_i)u_i(0) - G(b_i)u_i(0)(\frac{v_i - b_i}{\bar{v}}) + \\ & + u_i(0) - G(b_i)u_i(0)\} \\ & \max\{G(b_i)(v_i - b_i)(\frac{u_i(x)}{\bar{v}}) - G(b_i)(v_i - b_i)(\frac{u_i(0)}{\bar{v}}) + u_i(0)\} \\ & [\frac{u_i(x)}{\bar{v}} - \frac{u_i(0)}{\bar{v}}] \max\{G(b_i)(v_i - b_i)\} + u_i(0) \end{aligned}$$

It is clear that this problem is equivalent to that faced by a bidder in a risk-neutral environment, as all expressions involving the bidder's utility are constants.

In our experiment, a lottery is conducted after each round, where every subject has a chance to win an amount of 5 pounds. During such a round - 4 auctions, 2 in the role of Underdog and 2 in the role of Topcat - bidders should try to increase their chances in the lottery by maximising the profit they gain from auctions they

participate in. Profits are nominated in millions of dollars (as are valuations and bids), which are converted to percentages, (or chances for a win), in the lottery, according to the following conversion system: \$1m profit = 3 lottery tickets, and 10 lottery tickets = an additional 1% in the lottery. Bidders are also given an endowment of 10% (or \$33.3m) at the beginning of every round. All this information is presented to the subjects in the instructions to the experiment, before any strategic interaction begins.

5.2.1 Equilibrium Analysis

The analysis of the ascending price auction (treatment 1 in our experiment) is very straightforward in this setting. It is a dominant strategy for any bidder to keep bidding until the price reaches their true valuation for the licence. It is therefore clear that, in the setting described above, subjects playing the Underdog role can never gain any positive profit from participating in an ascending price auction, and thus should not agree to pay any positive fee in order to do so.

Optimal behaviour in the descending price auction is much more complex, and so we rely on an adaptation of the analysis of Vickrey's example, carried out in Appendix 3 of his 1961 paper. For generality's sake we ignore for the moment the parameter values used in the experiment, and adopt the general notation used in J. Swierzbinski's 2003 note on the equilibrium analysis of a first-price auction, in this case:

- Bidder 1's (Underdog) valuation v is uniformly distributed on the interval $[\omega_0, \omega_1]$.
- Bidder 2's (Topcat) valuation is fixed and commonly known to be equal to a where $a \leq 2\omega_1 - \omega_0$.

Based on Vickrey's assertion that the only equilibrium for the first-price auction in this setting involves bidder 2 using a mixed strategy and bidder 1 using an increasing single-valued bid function, we are looking for two objects: $F_2(x)$ - the c.d.f. describing bidder 2's equilibrium mixed strategy, and $\beta(v) = x$ - bidder 1's bid function which could be inverted to $\beta^{-1}(v) = \phi(x)$.

Also based on Vickrey's assertion we accept that the upper bound on both bidders' bids must, in equilibrium, be the same, i.e. both bidders will never bid above some x_{\max} , defined as $F_2(x_{\max}) = 1$, as any bid placed by bidder 1 above this value could be profitably reduced. We can also assume, without loss of generality, that the lower bound on both bidders' bids is the same, i.e. both bidders never bid below some x_{\min} defined as $F_2(x_{\min}) = 0$, as any bid of bidder 1 placed below this value can never win.

Given that bidder 2 is using a mixed strategy, he must be indifferent between all bids on the interval $[x_{\min}, x_{\max}]$, such that we have:

$$(a - x)F_1(\beta^{-1}(x)) = K_1$$

And since $F_1(\cdot)$ is the c.d.f. of the uniform distribution we have:

$$\begin{aligned}(a-x) \frac{\phi(x) - \omega_0}{\omega_1 - \omega_0} &= K_1 \\(a-x)(\phi(x) - \omega_0) &= K_1(\omega_1 - \omega_0) \\ \phi(x) &= \frac{K_1(\omega_1 - \omega_0)}{a-x} + \omega_0 \\ \phi(x) &= \frac{K}{a-x} + \omega_0\end{aligned}$$

where $K = K_1(\omega_1 - \omega_0)$. To get to bidder 1's equilibrium bid function we can substitute $\phi(x) = v$, and $x = \beta(v)$:

$$\begin{aligned}v &= \frac{K}{a - \beta(v)} + \omega_0 \\ \frac{1}{v - \omega_0} &= \frac{a - \beta(v)}{K} \\ \beta(v) &= a - \frac{K}{v - \omega_0}\end{aligned}$$

Vickrey argues that in equilibrium, the lowest of all of bidder 1's bids, that have some positive probability of winning, is the only time where bidder 1 bids his true valuation, i.e. that we have $\phi(x_{\min}) = x_{\min}$:

$$\phi(x_{\min}) = \frac{K}{a - x_{\min}} + \omega_0 = x_{\min}$$

Taking the derivative at x_{\min} we get:

$$\begin{aligned}\frac{d}{dx}\phi(x) \quad | \quad x=x_{\min} &= \frac{K}{(a - x_{\min})^2} = 1 \\ K &= (a - x_{\min})^2\end{aligned}$$

From the last two equation together we can get:

$$\begin{aligned}x_{\min} - \omega_0 &= \frac{(a - x_{\min})^2}{a - x_{\min}} \\x_{\min} &= \frac{a + \omega_0}{2}\end{aligned}$$

And using this we have:

$$\begin{aligned}K &= [a - (\frac{a + \omega_0}{2})^2] \\K &= \frac{a^2 - 2a\omega_0 + \omega_0^2}{4} \\K &= \frac{(a - \omega_0)^2}{4}\end{aligned}$$

Substituting the value of K into the inverse bid function $\phi(x)$ and the bid function $\beta(v)$ gives us:

$$\phi(x) = \frac{(a - \omega_0)^2}{4(a - x)} + \omega_0$$

And:

$$\beta(v) = a - \frac{(a - \omega_0)^2}{4(v - \omega_0)}$$

Bidder 1's bid function is strictly increasing on the interval $[x_{\min}, \omega_1]$, and thus it must be the case that: $\beta(\omega_1) = x_{\max}$ and $\phi(x_{\max}) = \omega_1$.

$$\begin{aligned}\phi(x_{\max}) &= \frac{(a - \omega_0)^2}{4(a - x_{\max})} + \omega_0 = \omega_1 \\(a - \omega_0)^2 &= 4(\omega_1 - \omega_0)(a - x_{\max})\end{aligned}$$

And so:

$$x_{\max} = a - \frac{(a - \omega_0)^2}{4(\omega_1 - \omega_0)}$$

To get to bidder 2's optimal mixed strategy we can use bidder 1's optimisation problem, and the functions $\phi(x)$ and $\beta(v)$. Bidder 1's problem is :

$$\max\{(v - x)F_2(x)\}$$

And the first order condition is:

$$\begin{aligned} -F_2(x) + (v - x)f_2(x) &= 0 \\ v - x &= \frac{F_2(x)}{f_2(x)} \end{aligned}$$

where $\frac{d}{dx}F_2(x) = f_2(x)$. Substituting $\phi(x) = v$ we have:

$$\begin{aligned} \phi(x) - x &= \frac{F_2(x)}{f_2(x)} \\ \frac{f_2(x)}{F_2(x)} &= \frac{1}{\phi(x) - x} \end{aligned}$$

But we know what the inverse bid function is:

$$\begin{aligned} \frac{f_2(x)}{F_2(x)} &= \frac{1}{\frac{(a-\omega_0)^2}{4(a-x)} + \omega_0 - x} \\ \frac{f_2(x)}{F_2(x)} &= \frac{4(a-x)}{(a-\omega_0)^2 + 4\omega_0(a-x) - 4x(a-x)} \\ \frac{f_2(x)}{F_2(x)} &= \frac{4(a-x)}{(a-\omega_0)^2 + 4\omega_0 a - 4\omega_0 x - 4xa + 4x^2} \\ \frac{f_2(x)}{F_2(x)} &= \frac{4(a-x)}{(a+\omega_0)^2 - 4x(a+\omega_0) + 4x^2} \\ \frac{f_2(x)}{F_2(x)} &= \frac{4(a-x)}{[2x - (a+\omega_0)]^2} \end{aligned}$$

Integrating this with respect to x we get:

$$\ln F_2(x) = -\ln[2x - (a + \omega_0)] - \frac{(a - \omega_0)}{2x - (a + \omega_0)} + C$$

Using the formula for x_{\max} and the fact that by definition $F_2(x_{\max}) = 1$ it is possible to solve for the value of the constant C :

$$C = \ln[(a - \omega_0)(\frac{2\omega_1 - \omega_0 - a}{2(\omega_1 - \omega_0)})] + \frac{2(\omega_1 - \omega_0)}{2\omega_1 - \omega_0 - a}$$

It is now possible to formulate the c.d.f that describes the optimal mixed strategy for bidder 2:

$$F_2(x) = \exp[\frac{2(\omega_1 - \omega_0)}{2\omega_1 - \omega_0 - a} - \frac{a - \omega_0}{2x - (a + \omega_0)}] \frac{2\omega_1 - \omega_0 - a}{2(\omega_1 - \omega_0)} \frac{a - \omega_0}{2x - (a + \omega_0)}$$

In order to be able to compare the equilibrium bidding and entry behaviour, of bidders in both the ascending and descending price auctions to that observed in our experiment, we need to give the parameters in the analysis above the values used in the lab, i.e. we need to substitute $\omega_0 = 0$, $\omega_1 = 100$, and $a = 100$. With these parameter values we have:

$$x_{\min} = 50$$

$$x_{\max} = 75$$

Bidder 1's equilibrium bid function is:

$$\beta(v) = 100 - \frac{10000}{4v} \quad (13)$$

And bidder 2's equilibrium mixed strategy is described by the c.d.f:

$$F_2(x) = \exp[2 - \frac{50}{x - 50}] \frac{25}{x - 50} \quad (14)$$

Figure 23 (panels a and b respectively) shows these two functions.

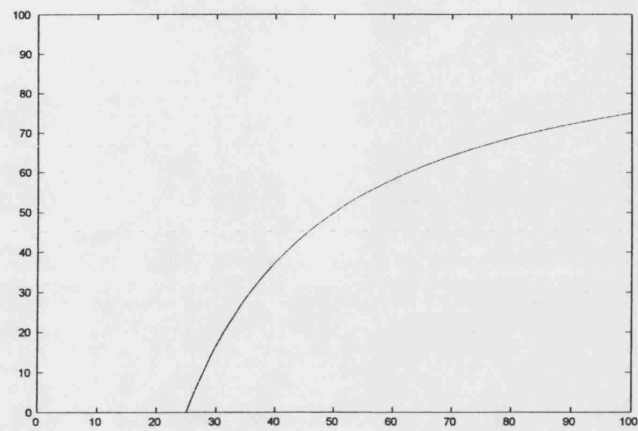


Figure 23a: Underdog's treatment 2 equilibrium bid.

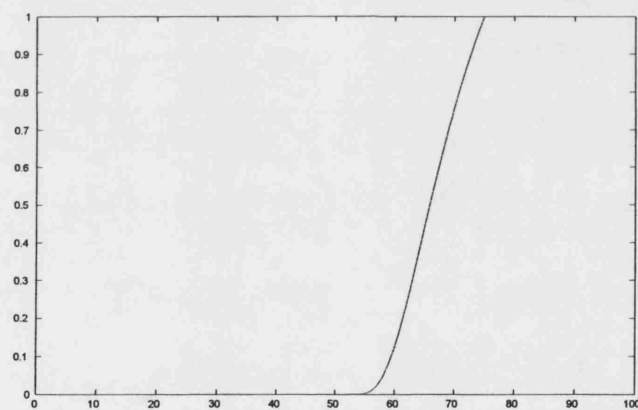


Figure 23b: Topcat's treatment 2 equilibrium cdf.

To see that this implies that the weak bidder has a positive probability of winning (which is absent in the ascending-price auction in this setting), and thus has an increased incentive to enter the auction, we can integrate the following expression

for bidder 1's expected probability of winning:

$$\int_{x_{\min}}^{\omega_1} F_2(\beta(v)) \frac{1}{\omega_1 - \omega_0} dv$$

taking into account that bidder 1 can never win with a valuation below $v = x_{\min}$.

Evaluating the expected probability of bidder 1 winning using the experimental parameter values we have:

$$\int_{50}^{100} \exp\left[2 - \frac{v}{v-50}\right] \frac{v}{200v - 10000} dv = 0.25 \quad (15)$$

This reveals that, in this setting, the weak bidder's chance of winning the auction is increased from 0% in the ascending-price auction to 25% in the first-price format.

We now need to calculate the expected profit bidder 1 makes when participating in the descending price setting, which will also serve as an upper bound on the entry fee that we can charge in our experiment. The weak bidder's expected profit in the first-price auction is:

$$\pi_1 = \int_{x_{\min}}^{\omega_1} (v - \beta(v)) F_2(\beta(v)) \frac{1}{\omega_1 - \omega_0} dv$$

Evaluated with our experimental parameter values we have:

$$\pi_1 = \int_{50}^{100} v - 100 + \frac{10000}{4v} \exp\left[2 - \frac{v}{v-50}\right] \frac{v}{200v - 10000} dv = 3.727 \quad (16)$$

Since we know that, in equilibrium, the weak bidder is unable to make any positive profit from participating in the ascending price auction, and thus will not be willing to pay any positive amount in order to do so, we can, in theory, separate the entry behaviour of weak bidders in the two auction formats completely, by charging an entry fee that lies in the interval $[0, 3.727]$. By 'complete-separation' in entry

behaviour, we mean that NO Underdogs in the ascending price auction should, in equilibrium, choose to pay in order to enter the auction, while ALL Underdogs should choose to do so in the descending price auction.

We therefore set the entry fee, that is applied in rounds 4-6 of the experiment, to 8 lottery tickets, which corresponds in our conversion system to an amount of \$2.66m, or 0.8% chance of winning the lottery. The entry fee was meant to be set in a way that balances two forces; it should be high enough for Underdogs in the ascending price treatment to ‘feel’ the loss of this amount if they do choose to enter, but low enough so that Underdogs in the descending price treatment should not become indifferent to entry altogether, and thus might prefer not to enter for reasons outside the experiment scope (like time considerations for example).

5.3 Results

We ran one experiment, i.e 6 rounds of 4 auctions (with 5 pairs of bidders in each auction bidding simultaneously), of each of the two treatments, as a pilot. The data from the pilot seemed to show a significant tendency of most subjects to follow the straightforward setup of the experiment, and behave, most of the time, in a seemingly rational way. This convinced us that the setup should not be changed, and we had two more runs done with the intent to use these as data if the tendencies remain the same. As they did, we ran three more experiment runs, and brought the total number of runs to five. The analyses reported below were thus conducted on 600 auctions in

each treatment (ascending or descending), 300 with free entry, and 300 with an entry fee.

We report the results in three different areas of interest; bidding behaviour, which enables us to view the data in light of the theory, and get an idea of the degree to which there is a fit between the two. Secondly we turn to the main interest of this research - the subjects' entry behaviour in the two auction formats. Here we hope to establish which of the two, ascending or descending price, (the latter is equivalent to the first-price sealed bid auction), is the more 'entry friendly' format in this setting. And lastly, we take a quick look at the revenues generated by these auctions, for a more complete picture with regards to policy considerations.

The experiment was built in a way that enables us to collect the information needed for these analyses; in the first half of the experiment participation in the auctions is free of charge and bidders get a chance to practice and learn the bidding game, this is where we take our bidding behaviour data from. Only after they have a good idea of what the game involves are subjects required to make the ex ante decision (in Underdog's case), of whether they wish to pay to participate in an auction before they know for sure their position in it. This is the focus of the second half of the experiment, where an actual auction doesn't always take place (whenever a bidder decides not to enter there is no auction), and we get our entry behaviour data.

5.3.1 Bidding Behaviour

In the experiment, both the ascending and descending auctions were conducted in the most straightforward way possible - at the beginning of the bidding a bar appears on the computer screen of every subject, with a scale running from 0 to 100, and an indicator indicating the current price of the licence. In the ascending price auction the price-indicator is gradually raised from zero, and stops either when one of the bidders chooses not to bid further (i.e. quits the auction), or when the lower of the two valuations is reached (i.e. Underdog's valuation), as no bidder is allowed to bid above his or her valuation. The auction then ends; the price is set at the value of the bid made, and the winner is announced to be the other bidder - the bidder who remained in the bidding - whose valuation and profit are also announced. In the descending price auction the price-indicator is lowered gradually from 100, above which no bidder is allowed to bid, until the auction is stopped by a bid. The price is set at the value of the bid, and the winner is the bidder who placed it.

Bidders were also allowed to set their bids in advance of the auction by using a slider provided for this purpose. In the ascending price auction, the 'slider-bid' serves as an upper bound on the subject's actual bid, as when an auction begins subjects can still stop the bidding at any point before the price reaches their own slider-bid. The default value for the slider-bid in the ascending price auction, if a subject chooses not to use the slider, is the bidder's own valuation. In the descending price auction a slider-bid serves as a lower bound on the subject's actual bid as bids could still

be placed above the value of the slider-bid, and the default value of that, in case no use of the slider was made, is zero. It is therefore clear that information on actual-bids is quite different from information on slider-bids. Generally the term 'bid' in what follows refers to 'actual-bid' made in the process of the auction, (any use of 'slider-bids' will be clearly indicated).

In the ascending price auction (treatment 1 in the experiment) our information on actual-bids includes only losing bids. Both bidders have the same dominant strategy of truthful bidding in this setting, so it is possible to pool losing Underdogs and losing Topcats together in comparing their bidding behaviour with the theory.

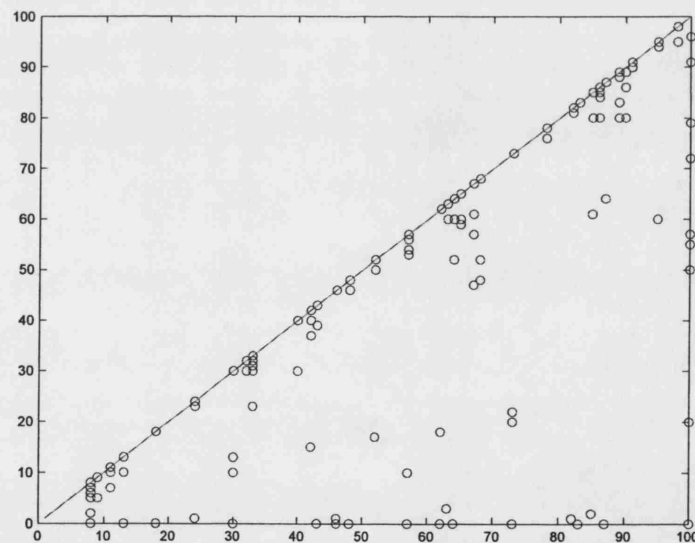


Figure 24: Losing bids in treatment 1.

Figure 24 shows a scatter-plot of all losing bids in treatment 1 (ascending price), rounds 2-3, as a function of the valuation of the bidder who placed them.

We exclude from the data the first round of bidding so that we look only at experienced bidders. Theory predicts that these bids should lie on the 45° line, which also appears in the figure. The figure shows a strong tendency of bidders to bid truthfully, with some underbidding, mainly by Underdogs.

Is this rational behaviour? Recall that it is a *weakly* dominant strategy for Underdogs to bid their true valuation, and in this setting, where they know they cannot win, the incentive to stay in the bidding until one's valuation is reached is even weaker. As for Topcats, their underbidding could cause them to lose an auction they would have otherwise won for sure, and thus their underbidding is irrational.

It is useful therefore to examine the number of times Topcats were actually hurt by underbidding, in the sense that it caused them to lose the auction. We find that the percentage of winners in treatment 1 who are Underdogs is 9.33%, (theory predicts that no Underdogs should win this auction). However, this figure is drastically reduced to 4.5% when we restrict attention to experienced bidders by excluding the first round of auctions.

In the descending price auction (treatment 2), it is no longer possible to look at Topcats and Underdogs together as they have very different equilibrium strategies. Our information on actual-bids in this treatment includes winning bids only.

In equilibrium, Topcats are using a mixed strategy, where they choose their bid from the interval $[50, 75]$ according to the exponential distribution described in equation 14. Figure 25 compares the c.d.f.s of the theoretical distribution, to that of

the empirical c.d.f, i.e. to observed actual-bids made by winning Topcats in rounds 2-3 of treatment 2.

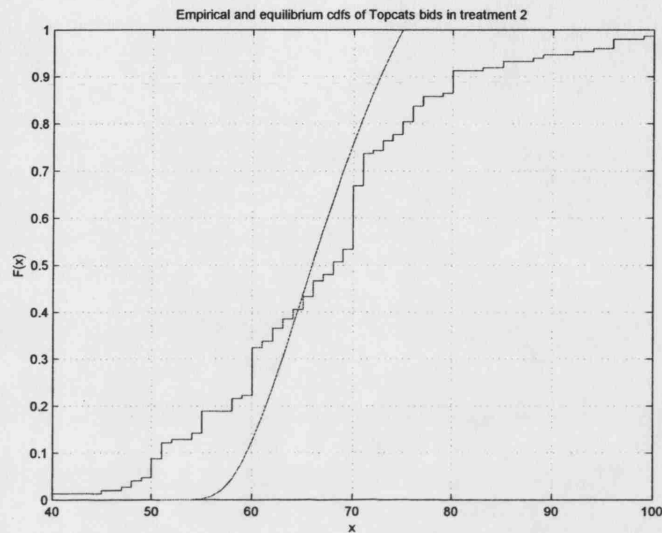


Figure 25.

In equilibrium of the descending price auction, Underdogs are using an increasing bid function as describes in equation 13. In Figure 26 we compare this equilibrium bid function to the scatter-plot of actual winning bids made by Underdogs in treatment 2 (rounds 2-3).

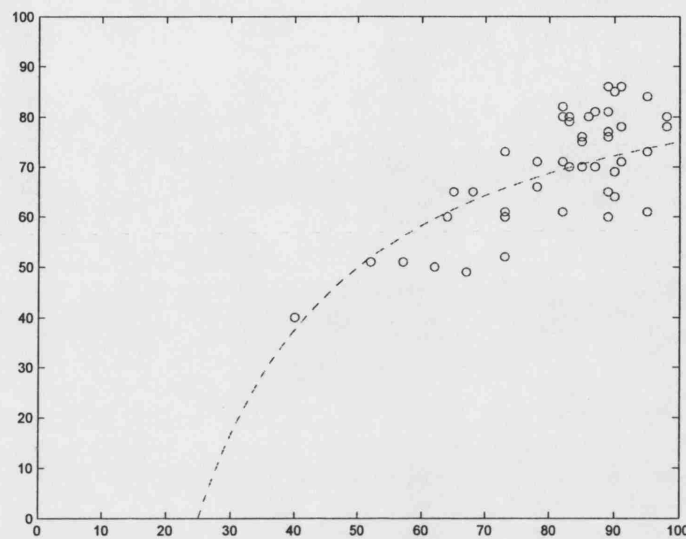


Figure 26: Underdog's observed and equilibrium bids.

Examining the last two figures reveals a slight tendency for overbidding by both Topcats and Underdogs in the descending price auction. It is interesting to see though, that when we look at the percentage of winners in treatment 2 who are Underdogs we find that it is staggeringly close to the prediction of the theory with 25% when all rounds of bidding are included, and 26% when we exclude round 1. Theory predicts that 25% of auctions should be won by Underdogs (see equation 15).

Overall, we assess the fit between the theory and the data on bidding behaviour in this setting, to be quite good. We now turn to the main focus of this experiment - the bidders' entry behaviour.

5.3.2 Entry Behaviour

In the second half of every experiment run - rounds 4-6 - bidders are required to pay to participate in each auction. Before they need to make this entry decision, they are informed of the amount of the entry fee (which was constant on 8 lottery tickets, or \$2.66m), and of the role they are going to play in this auction, if they choose to enter.

In the case of Topcats, whose valuation is fixed on \$100m, this is all the information they need in order to make this decision. Recall that the entry fee is set in such a way that it is always worthwhile for Topcats to enter the auction (both in the ascending and in the descending price formats). Underdogs however know at this point only that their valuation will be drawn from a uniform distribution on $[0, 100]$, so that after they decide to pay and enter an auction they might find that they have made a mistake. This, in our view, is a reflection of the situation telecom bidders, and especially potential newcomers who are faced by strong incumbents, often find themselves in.

Underdogs must therefore base their decision of whether or not to enter the auction on the expectation of their profits from doing so. Recall that the fee is set so that it is worthwhile for Underdogs in the descending price auction to pay the entry fee and participate in the auction, while the opposite is true for Underdogs in the ascending price auction (see equation 16).

We first want to look at the simplest representation of Underdogs' entry behaviour. Table 2 shows the percentage of Underdogs and Topcats paying the entry fee in both treatments.

		Auction format	
		Ascending	Descending
Bidder type	Underdogs	0.26	0.7667
	Topcats	1	0.9767

Table 2: Overall entry behaviour.

Although it is clear that there is a significant¹⁶ difference in entry behaviour of Underdogs in both treatments, there is also some entry occurring in treatment 1, where such behaviour seems irrational. However, Underdogs entering the ascending price auction are only behaving irrationally if they expect their Topcat opponents to adopt their own equilibrium strategy. We thus need a more refined, bidder-by-bidder analysis, that takes into account the history of each Underdog's encounters with Topcats when examining their entry behaviour. This analysis is carried out in Tables 3a-3e for the ascending price treatment, and in tables 4a-4e for the descending price treatment. Each table represent a single experiment run and contains a single row for each of the subjects who took part in that run. The tables compare for each

¹⁶ We tested the null hypothesis of the probability of entry in both auction formats being the same, against the hypothesis of that probability is larger in the descending price auction, using a single sided test of the differences of means of two binomial distributions (Takeshi Amemiya 1994). We rejected the null hypothesis in the confidence level $p = 0.01$.

subject in the Underdog role the average profit gained by this subject up to every entry decision-point, to their choice of entry at that point (a more detailed account of the information in the tables is brought below).

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	0	0	0	1	0	1	0	0.667
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0
5	13	0.5	11.142	0	11.142	0	11.142	0.166
6	0	0.5	0	0	0	0	0	0.166
7	2	1	1.5	0	1.5	0	1.5	0.333
8	7.166	0	7.166	0	7.166	0	7.166	0
9	8	1	11	1	8.8	1	7.333	1
10	10.333	1	8.125	1	6.5	1	5.416	1

Table 3a.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	1	0	1	0	1	0	1
4	0	0	0	0	0	0	0	0
5	9	0	9	0	9	0	9	0
6	3.166	0	3.166	0	3.166	0	3.166	0
7	0.166	0.5	0.142	0	0.142	0	0.142	0.166
8	6.667	0	6.667	0	6.667	0	6.667	0
9	0	0	0	0	0	0.5	0	0.166
10	0	0	0	0	0	0	0	0

Table 3b.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	0	0.5	0	0	0	0	0	0.166
2	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0.333
4	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	0.333
6	1.667	0.5	1.428	0.5	1.269	0	1.269	0.333
7	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0
9	5.667	0	5.667	0	5.667	0	5.667	0
10	0	0	0	0	0	0	0	0

Table 3c.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	0	0	0	0	0	0	0	0
2	2.333	1	1.75	1	1.4	1	1.166	1
3	0	1	0	0	0	0	0	0.333
4	0.333	0	0.333	0.5	0.296	0.5	0.269	0.333
5	14.667	0.5	12.571	0.5	11.174	0	11.174	0.333
6	7.333	0	7.333	0	7.333	0	7.333	0
7	0	0	0	0	0	0	0	0
8	0	0.5	0	0	0	0	0	0.166
9	0	0	0	0	0	0	0	0
10	0	1	0	1	0	1	0	1

Table 3d.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	5.667	0	5.667	0	5.667	0	5.667	0
2	0	0.5	0	0	0	0	0	0.166
3	0	0	0	0	0	0	0	0
4	6.333	1	4.75	1	3.8	0.5	3.454	0.833
5	14	1	10.5	1	8.4	1	7	1
6	4.166	0	4.166	0	4.166	0	4.166	0
7	4.5	1	5.875	1	4.7	1	3.916	1
8	12.5	0	12.5	0.5	11.111	0	11.111	0.166
9	3.166	0.5	2.714	0	2.714	0	2.714	0.166
10	0	0	0	1	0	1	0	0.667

Table 3e.

Tables 3a-3e report the entry behaviour of bidders in the Underdog role in the ascending price treatment, tables 4a-4e below show the equivalent information for Underdogs in the descending price treatment.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	1.833	1	1.375	1	1.1	1	1	1
2	0	0.5	0	0	0	0	0	0.166
3	3.166	1	3.25	0.5	2.889	1	3.657	0.833
4	0	0.5	0	0	0	0	0	0.166
5	1.833	0.5	1.571	0	1.571	0	1.571	0.166
6	1.833	1	2.625	1	3.3	1	2.75	1
7	2	0.5	1.714	1	1.371	0	1.371	0.5
8	2.833	1	2.125	0.5	1.889	0.5	1.717	0.667
9	0	0	0	1	1.5	1	1.25	0.667
10	1.833	1	1.375	1	1.1	1	0.916	1

Table 4a.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	5.833	0.5	5.714	0	5.714	0.5	5.922	0.333
2	2.5	1	1.875	1	3.5	1	3.583	1
3	0.833	0	0.833	1	0.667	1	1.805	0.667
4	3.333	1	4.625	1	3.7	1	4.75	1
5	10.833	1	11.25	1	9	0.5	9.272	0.833
6	4.333	0	4.333	1	3.866	1	3.222	0.667
7	0	0	0	0	0	1	2.916	0.333
8	4.833	1	3.625	1	4.1	1	4.083	1
9	0	0.5	0	0.5	0	0	0	0.333
10	8.5	1	6.375	1	6.8	0.5	6.181	0.833

Table 4b.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	0	0	0	1	0	0	0	0.333
2	0	0.5	0	1	1	0.5	1.545	0.667
3	5.667	1	6.125	1	4.9	0	4.9	0.667
4	3	1	3.875	1	3.1	1	3.75	1
5	6.667	1	7.25	1	6.2	1	6.833	1
6	5.667	1	5.625	1	4.5	1	3.75	1
7	3.333	0.5	4.714	1	4.171	1	5.392	0.833
8	0	0	0	0	0	0.5	0	0.166
9	3.166	1	2.375	0	2.375	1	1.979	0.667
10	0	1	0	1	1.7	1	1.416	1

Table 4c.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	4.667	1	3.5	1	2.8	1	2.583	1
2	2.333	1	1.75	1	3.6	1	4.166	1
3	4	1	4.875	1	3.9	1	5.333	1
4	3	0.5	4.142	0.5	3.682	0.5	3.347	0.5
5	2.667	1	6	1	4.8	1	4.75	1
6	0	1	2.75	1	2.2	1	1.833	1
7	3.5	1	2.625	1	2.1	1	1.75	1
8	6.667	1	9.125	1	7.3	1	6.083	1
9	2	0.5	1.714	1	3.871	1	3.226	0.833
10	7.833	1	5.875	1	7.4	0.5	6.727	0.833

Table 4d.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	3.333	1	2.5	1	2	1	1.667	1
2	1.5	0.5	1.285	1	2.428	1	2.023	0.833
3	0	1	3.75	1	3	1	4.5	1
4	4	1	4	1	3.2	1	3	1
5	4.5	1	4.75	1	3.8	1	3.166	1
6	2.166	1	2.625	1	2.7	1	2.25	1
7	0	0.5	0	1	0	1	2	0.833
8	6.333	1	4.75	1	3.8	1	3.166	1
9	0.166	0	0.166	0	0.166	0	0.166	0
10	3	1	2.25	1	3.3	1	2.75	1

Table 4e.

The tables above (3a-3e and 4a-4e) report for each subject in the Underdog role (subject number in column 1), in both treatments of the experiment, the average profit gained by that bidder up to every entry decision-point, and compare that to the entry decision made at that point. The first entry decision-point in the experiment occurs in round 4, where the entry fee is introduced. The second column of the table therefore shows the average profit each bidder gained, in auctions he or she played

the Underdog role, up to round 4. In round 4, (as in all other rounds) each subject will play the Underdog role twice, so that when we look at a bidder's entry behaviour in round 4 we find one of three outcomes: 0 entries, 1 entry out of two, or 2 entries, the corresponding entry behaviour would be reported as 0, $\frac{1}{2}$ or 1 respectively. This information appears in the 'Entry in round 4' column (column 3), and similarly for every other entry decision-point i.e. in rounds 5 and 6 (columns 5 and 7 respectively). The fourth column shows the average profit gained by each bidder in the Underdog role up to round 5. This figure will, of course, depend on that bidder's entry decisions in round 4, and in particular, it will be unchanged if the bidder had 0 entries in round 4. That is, when a bidder chooses not to enter an auction, he receives no additional information that can affect his behaviour at the next decision-point and we treat these auctions as if they did not take place, when calculating this bidder's average profit up to his next decision-point. This is, of course, different from the case of a bidder choosing to enter the auction, and then making zero profit upon entering and bidding in the auction, this bidder's average profit will now be reduced, (as the same amount of profit is now divided by a larger number of auctions).

All average profits are reported in millions of dollars, and are *inclusive of the entry fee* that is, the entry fee is not deducted from the profits earned in any of the auctions, and thus they never appear in the table to be negative. The monetary amount with which these average profits should be compared in order to assess whether entry is worthwhile, is the amount of the entry fee, i.e. \$2.66m. Whenever the average

profit gained by a bidder in the role of Underdog is lower than the cost of entry, that bidder should not, given the information at their disposal, enter the next auction, and thus all the auctions that follow (when they play the Underdog role).

The last column in each table (column 9) reports the overall entry behaviour of each bidder, i.e. the proportion of all auctions (in rounds 4-6) in which this bidder chose to participate. Column 8 shows the average profit gained by each bidder over all auctions in the experiment, where he played the Underdog role and chose to participate.

Tables 5 and 6 have the equivalent entry behaviour analysis carried out for subjects playing the Topcat role in both treatments. Tables 5a-5e report entry behaviour of Topcats in the ascending price treatment.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	57.5	0.5	63.571	1	61.657	1	63.131	0.833
2	12.667	0.5	14.571	1	30.157	1	38.964	0.833
3	29.5	1	37.125	1	46.3	1	54.583	1
4	44.166	1	43.625	1	54.9	1	60.916	1
5	54.833	1	64.75	1	62.9	1	69.083	1
6	48.833	1	61.625	1	69.3	1	70.416	1
7	59.333	1	57.625	1	64.9	1	70.75	1
8	62.5	1	71.875	1	77.5	1	76.333	1
9	39.833	1	54.875	1	60.6	1	67.166	1
10	43.833	1	57.875	1	66.3	1	71.916	1

Table 5a.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	92.5	1	94.375	1	95.5	1	94.667	1
2	47.333	1	60.5	1	68.4	1	73.667	1
3	47.833	1	60.875	1	68.7	1	????	1
4	68.5	1	76.375	1	81.1	1	84.25	1
5	54.833	1	66.125	1	72.9	1	77.416	1
6	57	1	67.75	1	70.3	1	75.25	1
7	77.166	1	73.125	1	78.5	1	82.083	1
8	66.833	1	75.15	1	80.1	1	79.25	1
9	49.5	1	62.125	1	66.5	1	72.083	1
10	47.5	1	50.625	1	60.5	1	66.33	1

Table 5b.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	71.333	1	78.5	1	82.8	1	85.667	1
2	42.833	1	57.125	1	58.4	1	65.333	1
3	36.833	1	42.75	1	54.2	1	61.833	1
4	61.166	1	70.875	1	76.7	1	80.583	1
5	70.5	0.5	74.714	1	79.771	1	83.142	0.833
6	51.333	1	63.5	1	62.6	1	68.833	1
7	61.833	1	71.375	1	77.1	1	80.916	1
8	59.667	1	62.75	1	66.4	1	72	1
9	35.667	1	51.75	1	58.2	1	65.166	1
10	51	1	63.25	1	70.6	1	75	1

Table 5c.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	67.5	1	75.625	1	71.3	1	76.083	1
2	36	1	51.625	1	61.3	1	64.916	1
3	43.833	1	57.875	1	62.9	1	69.083	1
4	45.667	0.5	53.428	1	62.742	1	67.452	0.833
5	47.166	1	52.75	1	62.2	1	68.5	1
6	58	1	61.5	1	64.9	1	70.75	1
7	63.333	1	72.75	1	78.2	1	75.166	1
8	74.833	1	81.125	1	75.9	1	79.916	1
9	33.5	1	45.75	1	56.6	1	56.667	1
10	42	1	56.375	1	58	1	65	1

Table 5d.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	61.667	1	71.25	1	60.6	1	58.833	1
2	45	1	49.5	1	59.6	1	63.5	1
3	22.667	1	42	1	44.1	1	53.416	1
4	43.333	0.5	37.714	1	50.171	1	56.309	0.833
5	54.166	1	65.625	1	68.9	1	74.083	1
6	65.5	1	65.625	1	68.4	1	73.667	1
7	66.166	1	65.25	1	71.9	1	70.75	1
8	53.667	1	54.875	1	60.3	1	66.916	1
9	39.833	1	54.375	1	63.5	1	66.083	1
10	60	0	60	1	66.8	1	67.333	0.667

Table 5e.

Tables 6a-6e carries out the same analysis of entry behaviour for Topcats in the descending price treatment.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	28.667	1	34	1	32.7	1	38.083	1
2	25	1	37.5	1	35	1	41.667	1
3	15.833	1	16.875	1	17.5	1	20	1
4	22	1	34	1	41.2	1	46.833	1
5	18	1	26.75	1	27.9	1	28.667	1
6	13.333	1	26.875	1	41.5	1	45.583	1
7	24.667	1	31	1	39.1	1	49.25	1
8	22.5	1	33.125	1	46.5	1	38.75	1
9	11.333	1	11	1	12.2	1	26.833	1
10	6.5	1	6.5	1	25.2	1	29.5	1

Table 6a.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	43.166	1	50.875	1	40.7	1	42.25	1
2	25.166	1	35.625	1	33	1	35.833	1
3	10.5	1	22.875	1	32.2	1	37.667	1
4	26.5	1	44.875	1	40.8	1	41.667	1
5	23.833	1	28.5	1	42.8	1	38.166	1
6	17.5	1	13.125	1	17.5	1	22.916	1
7	31.667	1	36.25	1	43	1	35.833	1
8	34	1	25.5	1	23.8	1	19.833	1
9	23.333	1	25.625	1	35.8	1	34.166	1
10	22.333	1	41.75	1	36.8	1	40.667	1

Table 6b.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	22.333	1	19.75	1	25.8	1	23.583	1
2	24.5	1	23.75	1	32	1	37.416	1
3	9	1	24.125	1	23.1	1	27.166	1
4	30.333	1	37.75	1	44.7	1	46	1
5	21.5	1	33.25	1	41	1	42.583	1
6	21	1	28.25	1	31.7	1	34.75	1
7	29.333	1	22	1	26	1	21.667	1
8	33.833	1	42.625	1	34.1	1	45.083	1
9	22.166	1	27.625	1	32.1	1	36.75	1
10	26.5	1	36	1	34.7	1	39.25	1

Table 6c.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	25.5	1	31.625	1	27.9	1	27.416	1
2	25.333	1	29.125	1	30.5	1	31.833	1
3	19.166	1	14.375	1	20	1	22.916	1
4	27.5	1	27.5	1	31.1	1	38.75	1
5	36.833	1	39.125	1	36	1	37.833	1
6	23.166	1	34.5	1	32.9	1	37.916	1
7	28.833	1	21.625	1	24.7	1	23.583	1
8	31.333	1	29	1	27.3	1	22.75	1
9	23.833	1	25.75	1	27.9	1	27.583	1
10	17.166	1	21.375	1	31.4	1	28.416	1

Table 6d.

Subject No.	Average profit up to round 4	Entry in round 4	Average profit up to round 5	Entry in round 5	Average profit up to round 6	Entry in round 6	Overall average profit	Overall entry proportion
1	30	1	35	1	32.8	1	35.667	1
2	18.833	1	22.625	1	28.1	1	32.416	1
3	20.667	1	22.375	1	24.9	1	32.416	1
4	31	1	47.5	1	45	1	44.166	1
5	21.166	1	25	1	35.8	1	37.083	1
6	22.833	1	22.75	1	26.2	1	25.166	1
7	23.667	1	17.75	1	20.3	1	21.5	1
8	12.5	1	14.375	1	14.2	1	14.333	1
9	19.5	1	28.275	1	26.1	1	25.083	1
10	10.166	1	8.875	1	12.9	1	18.166	1

Table 6e.

Possible Source for Seemingly Irrational Entry Behaviour

The entry tables above shed some light on the process governing the bidders' decision making regarding entry. It seems that, in the case of the ascending price auction, an encounter in the first three rounds, of an Underdog with a Topcat opponent that yields profit for Underdog, is likely to increase the chances of Underdog choosing to enter in the later rounds. Look, for example, at bidder number 9 in table 3a. Up to round 4, this bidder gained an average profit of 8m, when he played Underdog - well above the entry fee. In round 4 he then chooses to enter twice, and he managed to increase his average profit further to 11m. From that point on bidder 9 chooses to enter every time he plays Underdog, although his average profit keeps dropping. His overall average profit is 7.333m - still well above the entry fee. Is bidder 9 behaving irrationally?

Such high profits for Underdog in the ascending price auction are, however, an evidence that Topcat was not following his equilibrium strategy. In fact it is an evidence that Topcat was underbidding by so much that he placed his bid before the price reached Underdogs valuation, since Topcat could, in some cases, underbid and still win the auction, if Underdog's valuation is lower still, (or at least he could intend to underbid, as we do not have information on actual bids of winners in the ascending price auction).

If Underdog places some positive probability on that event occurring, then it might be worth his while to enter the ascending price auction even when doing so is costly. Underdog might believe that Topcat's hand is trembling, and thus he might make a mistake and place his bid earlier than intended.

What would Underdog have to believe about Topcat's trembling hand in order to justify entry in the ascending price auction? The larger is Topcat's bidding mistake, the larger is the potential profit Underdog could make when facing him, and thus the smaller is the probability of this mistake occurring that would justify Underdog's entry. So, Underdog would have to believe that Topcat's actual bidding behaviour (taking into account his trembling hand) follows some exponential probability distribution, that places very small probabilities on very small bids, with larger and larger probabilities on bids as they get closer to 100 - Topcat's valuation. Every probability distribution that satisfy the following equation:

$$\int_0^{100} \int_0^{100} (v - x) \frac{f(x)}{100} dx dv \geq 2.66$$

where v is Underdog's valuation which we know is distributed uniformly on $[0, 100]$, x is Topcat's bid, and $f(x)$ is the probability distribution that describes Topcat's bidding behaviour, would justify Underdog's entering the auction.

It is obviously impossible to assess whether the subjects playing the Underdog role, who chose to enter the ascending price auction, did in fact hold such beliefs regarding Topcats bidding behaviour that would justify them entering. It is however possible to say that the size of the entry fee, that is limited to a rather small amount by the need to keep entry into the descending price auction profitable, is such that the probability for a mistake on Topcat's part need not be enormous in order to justify entry into the ascending price auction as well.

5.3.3 Auctioneer's Revenue

Lastly we want to look briefly at the revenue each of the two auction formats used in our experiment generate for the auctioneer. Revenue is not the focus of this work, since as we saw in the previous chapters, it is sometimes necessary to make sacrifices in terms of efficiency and revenue in order to promote entry.

It is well known that the revenue equivalence breaks down in the presence of bidder asymmetries, but our simple setting makes it quite straightforward to determine which of the two auction formats dominates in terms of revenue.

In the ascending price auction, where the auction clearing price is set at the losing bid, and where bidders' dominant strategy is truthful bidding, our simple set-

ting makes it clear that the expected auction price is simply the expected valuation for Underdog, or 50.

In the descending price auction, as always, things are more complex; it is however quite simple to show that the expected auction price must lie above the 50 mark. All that is needed is to recall both bidders equilibrium strategies; Topcat uses a mixed strategy in equilibrium, where he chooses his bids from the interval $[50, 75]$ according to some probability distribution function (described above). Underdog follows an increasing equilibrium bid function (also fully described above), in which we restricted his bids to have the same upper and lower bounds as those of Topcat, i.e. Underdogs bids also belong to $[50, 75]$. The upper restriction (75) is necessary for the equilibrium to exist, while the lower restriction (50) is placed without loss of generality, since as Vickrey noted, the bidder in the underdog role can never win with a bid below that value.

It is therefore clear that, in the descending price auction, where the auction price is determined by the winning bid, no winning bid could ever be placed below 50, while they could get as high as 75. We can thus conclude that, in the setting described above, the expected price in the descending price auction is higher than that in the ascending price auction.

The data from the experiment support this prediction. Figures 27 and 28 show a scatter-plot of the auction-clearing price as a function of Underdogs' valuation in the ascending and the descending price auctions respectively. Recall that, although

within every treatment in the experiment Underdog's valuation in every auction is a new draw from the same distribution, valuations are the same between the two treatments.

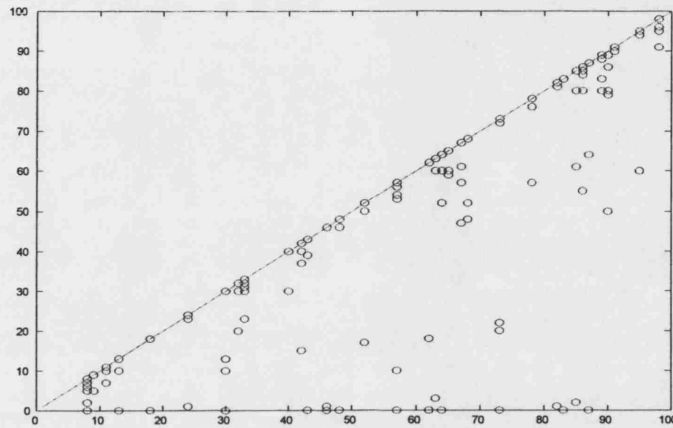


Figure 27: Auction clearing price in treatment 1.

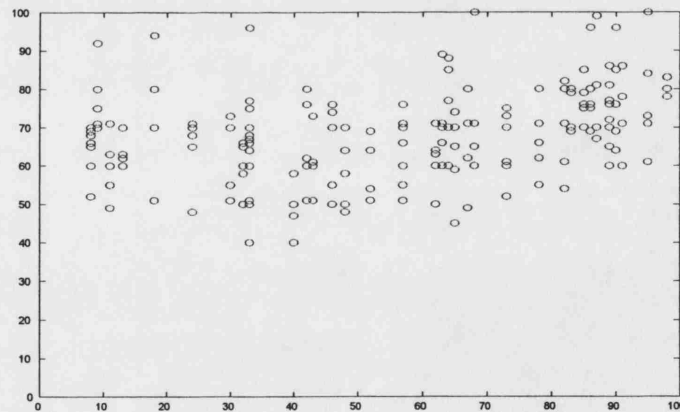


Figure 28: Auction clearing price in treatment 2.

The average clearing prices in the two auction formats are 45.97 in treatment 1 (ascending price), and 67.66 in treatment 2 (descending price).

5.4 Conclusion

Our experiment was designed to test the properties of two of the most common auction formats - the ascending and descending price auctions - in a controlled environment. We are especially interested in the attractiveness as an investment of each one of these formats, to bidders who are known to be weaker than their opponents. Entry behaviour of the subjects is therefore the main object of our analyses, while bidding behaviour serves as a test for the goodness fit between our data and the theory, and thus for the validity of the data for drawing conclusions with regards to entry.

The bidding behaviour data suggest that bidders in our experiment understood the task and the environment they operate in. In most cases they learnt how the mechanism works and what needs to be done in order to increase their profits, and acted accordingly. Their entry decisions, made in the second part of the experiment are therefore viewed as driven of the same profit-maximising desire, and thus are reflective of the entry promoting properties of the two auction formats.

Given the analyses presented above we can conclude that the data supports our initial conjecture - *in an asymmetric environment, were some bidders are known to be ex ante weaker than others, the descending price format is more promotional of weak bidders' participation than the ascending price format. In the case were*

both auctions are equally costly, ex ante weak bidders choose to participate in a descending price auction significantly more often than they do in the ascending price format.

5.5 Related Literature

The experimental literature dealing with asymmetric auctions is not large despite, or perhaps because of the fact that a theoretical treatment of these auctions is very complex, and often intractable. There are, however, several papers that do address the properties of asymmetric auction in a controlled laboratory environment. Many of these experiments, like that of Abbink et.al. (2001) mentioned in the introduction to this chapter, were designed to test a specific auction format for the purpose of advising either an auctioneer or a bidder on the optimal behaviour in a real world auction situation, where asymmetries among bidders are important. These papers often address the question of bidder participation, or entry in their experimental settings.

Abbink et.al. (2001) compare the performance of three auction formats, all of which were mentioned at some point as a possibility for serving as the UK 3G auction. Two of the auctions are of the two-stage mechanism type often referred to as the ‘Anglo-Dutch’ auction. In both, the first stage consists of an open, ascending, bidding stage which ends when the number of bidders remaining is equal to the number of items on sale (4 in their experiment) plus one. Both then proceed to a second-stage where all bidders submit a single sealed bid for a single item. The winners are the

four bidders whose bids are the highest, but the payment rule is different in the two auction formats. One uses the *uniform* pricing rule, where all winners pay the same price equal to the 4th highest bid, and the other uses the *discriminatory* pricing rule where each winner pays his own bid. Both these auctions were compared to the simple English auction. Bidders in their experiment were asymmetric. Valuations for each item were constructed of a common and a private components. The common component is the same for all bidders, but unknown to them with certainty. Instead they each received a private signal on this common value, all signals were drawn from the same distribution around the true common value. Bidders also had a private component to their valuation, which was known to each bidder privately. The distribution of the private component of valuation depends on the type of the bidder - INC or NEW - for incumbents and new entrants respectively. Incumbents had an 80% chance that their private value will be drawn from a positive distribution, and 20% chance that it will be drawn from an equivalent negative distribution. The opposite was true for new type bidders.

The results of their experiment are reported in four different fields; total surplus, or the sum of the winners valuations, as a measure of efficiency, the number of successful new type bidders, the auction revenue, and the avoidance of the winner's curse. Only very small (mostly insignificant) differences could be detected with respect to these criteria, and those seem to vanish when bidders become experienced.

Most related to our experiment is the analysis given in Abbink et.al. (2001) to the issue of entry. The measure of entry use in their experiment is "the number of successful new-type bidders" (p.13) 'New-type bidders' correspond in their setting to the disadvantaged potential entrants, who are ex ante, (but not necessarily ex post) weaker than the incumbents. The important difference though is that they restrict their attention to "successful" entrants, that is, to entrants who succeed in winning the auction. We, on the other hand look (throughout this work), at the number of new entrants bidding in the auction, as we see the auction format as best serving the entry goal by making sure that potential entrants are not deterred from participating in the bidding in the first place (see chapter 1 for a full analysis of the concept of 'entry' adopted in this work). With respect to their measure of entry Abbink et.al. find that "the highest number of successful new entrants can be found in the English auction", furthermore, the difference between the English and the discriminatory auctions performance in 'entry' is found to be weakly significant, while between the English and uniform is was found not to be significant.

Guth, Ivanova-Stenzel, and Wolfstetter (2001) analyse an environment where valuations are private information and independently drawn from two distinct but commonly known distribution functions. In particular they use two uniform distributions, the weak bidder's valuation is drawn from the interval $[50, 150]$ and the strong bidder's valuation belongs to $[50, 200]$. Pairs of bidders, one of each type, participate in a two-bidder bidding competition for a single item in both a first-price and

a second-price auctions. They compare the revealed properties of the two auction formats to the theory prediction. Most related to our approach, they test, by letting bidders pay in order to dictate the auction format they participate in, which of the two auctions is preferred by the bidders.

They find that, contrary to theory prediction, which says that average profit for the weak bidder is higher in the first-price auction, both bidders have a significantly higher average profit in the second-price auction. Both bidders types thus choose to participate in the second-price auction overwhelmingly more often than the first-price auction, when they are given the chance to dictate the format. Most bidders are, however, willing to pay for the dictatorship right less than would be justified by the difference in average profits.

Goeree, Offerman, and Schram (2003) compare the performance of three auctions, all of which have a first-price element in their format to the more commonly used simultaneous ascending price auction. The three first-price auctions are; the simultaneous first-price (sealed bid) auction, the sequential first-price (sealed bid) auction, and the simultaneous descending (open) auction. They look at the performance of the different formats in several areas, of which the most relevant for us is the promotion of entry. In order to test this feature they construct one part of the experiment (part 4) such that in it 8 bidders, of which 3 are strong incumbents, and 5 are weak newcomers, have the opportunity to compete for three telecom licences of varying qualities. The potential entrants (the weak bidders) also have an outside op-

tion in the form of a lottery they can take part in, which bears some positive expected profit. Before the auction begins every entrant has to decide whether or not he/she wants to participate in the auction, or take part in the lottery.

Their results indicate that there is a positive correlation between average profits gained in early parts of the experiment and bidders entry behaviour in part 4. The simultaneous descending and simultaneous first-price auctions yielded the highest profit for newcomers, and thus induced the highest entry, third is the simultaneous ascending auction, and the lowest level of entry was observed in the sequential first-price auction, where average profits were also the lowest. It is interesting to note, however, that in all four auction formats entry was found to be higher than would be efficient when taking expected profits into account, and in that respect it is the sequential first-price auction that performed well with the lowest level of over-entry.

Athey, Levin, and Seira (2004) conduct a natural experiment, to compare the performance of the open ascending auction to the sealed bid first-price auction in an asymmetric environment. They compare the two formats in terms of both participation of weak bidders and the revenue they generate. The data is taken from the US Forest Service timber auctions conducted in two regions in the US, where historically both auction formats were used to sell tracts of timber. Weak bidders in the model are identified as bidders with no manufacturing capacities, who they refer to as 'loggers', and whose only option is to resale the timber. The strong bidders are 'mills' - companies that can either sell the timber or use it in production.

The authors find that first-price sealed bid auctions induce significantly more participation by loggers, who also go on to win more often in the sealed bid format. Revenues are found to be significantly higher in the first-price sealed bid auctions in one of the two regions in their sample, while in the other region differences in revenues were small and could not be statistically distinguished from zero.

Ivanova-Stenzel, and Salmon (2003) examine entry behaviour in a symmetric environment, where all valuations are drawn from a uniform distribution on $[0, 100]$. In the experiment each subject first participate in both a first-price sealed bid auction, and an open ascending auction, with the same valuation and facing the same (single) opponent (the results from the first-price auction were revealed only after both auctions were over). Then subjects were asked to choose which auction they want to pay in order to participate in, where, in the first treatment, both auctions had the same entry fee. Finally the authors investigate the bidders willingness to pay for participating in their 'favourite' auction format.

Their results indicate an overwhelming preference for the ascending price auction when entry fees for both are equal. However, the willingness to pay for participating in the ascending auction was found to be much lower than would be justified by the differences in average profits gained in each auction format.

There are other experimental studies that examine endogenous entry in auctions, however, most of these are based in a symmetric setting and do not involve a

comparison of that behaviour between different auction formats. We look at some of these studies briefly.

Palfrey and Pevnitskaya (2003) investigate a symmetric environment, where bidders to choose whether to enter a first-price sealed bid auction or collect a fixed outside option. An alternative auction format was not investigated, but the setting was changed by varying the value of the outside option with respect to the highest possible valuation a bidder could draw (which was common to all bidders), the number of potential bidders in an auction was also variable. These variations would create different incentives for entry, and in particular the theoretic model used by Palfrey and Pevnitskaya as a benchmark predicts that entry frequencies should be lower than that in the risk-neutral equilibrium levels, which would suggest heterogeneous risk attitude among the bidders.

The authors find that observed entry frequencies were actually significantly higher than predicted by the risk-neutral equilibrium. On the other hand, bidding behaviour in auctions where entry was voluntary was less aggressive than in auctions with a fixed number of bidders, which could be attributed to more risk-averse bidders choosing not to enter. As predicted, entry frequencies were found to decrease with the value of the outside option.

As no comparison to an open ascending auction was carried out, these results, although interesting, bear little relevance to our experimental analysis, where the

possibility of risk attitude affecting bidding or entry behaviour was neutralised by the use of lotteries.

A different approach is taken in Bajari and Hortacsu (2000), where the authors use data from auctions conducted on the internet site eBay to perform a natural experiment. They address several issues that arise in the unique setting of eBay, such as secret vs. announced reservation price, but they also look at the question of entry into these auctions. In their setting bidders can choose among several auctions in which roughly the same object is being sold. The auction formats are also very similar, and vary only within the eBay basic guideline.

In this setting the authors find that attracting entry into the auction is one of the important determinants of the seller's revenues. They find evidence that suggests that entry to these auctions is costly and that bidders tend to choose the auction where they expect to have the best chance of winning.

Appendix A

A possibility for a loss in the telecom market

We now consider the case where, in the model of chapter 3, the winner of the telecom auction may find out after acquiring a license that there is no real opportunity for positive profits in the market, and therefore decide to go out of business. This, not unrealistic, scenario means in terms of this model that after the auction the value of Δ is revealed to be below some threshold that insures a non-negative profit. The winner in this case should choose optimally to produce a zero amount. It is important to note the timing of the game in this case:

1. Entrant decide on entry.
2. Entrant learns his type.
3. Bidders bid in the auction.
4. The winner of the auction pays $F(r)$ to the auctioneer, where r is the effective royalty rate.
5. Δ is revealed.
6. The winner decides how much to produce.

A.0.1 The Market

Assume that the inverse demand function takes the form: $p = (\frac{\Delta}{q})^{\frac{1}{2}}$ where as before Δ follows a Gamma distribution with $f(\Delta) = \frac{1}{m!} \Delta^m e^{-\Delta}$, $\mu(\Delta) = m + 1$, and $\sigma^2(\Delta) = m + 1$.

Similar to the analysis in chapter 3, whoever wins the auction of the second stage and operates in the market can make a profit of:

$$\pi = q(\frac{\Delta}{q})^{\frac{1}{2}}(1 - r) - cq - G$$

where the fixed fee $F(r)$ is regarded as sunk and $G \in \{I, E\}$ is the winner's cost.

Profit maximization requires:

$$\begin{aligned} \frac{d\pi}{dq} &= \frac{1}{2}(1 - r)(\frac{\Delta}{q})^{\frac{1}{2}} - c = 0 \\ q^* &= \Delta(\frac{1 - r}{2c})^2 \\ p^* &= \frac{2c}{1 - r} \\ R^* &= \Delta(\frac{1 - r}{2c}) \end{aligned}$$

where * represent optimal values.

It is now possible to calculate the maximal profit which is a random variable before Δ is revealed:

$$\begin{aligned} \pi_{\max} &= \Delta(\frac{1 - r}{2c})(1 - r) - c\Delta(\frac{1 - r}{2c})^2 - G \\ \pi_{\max} &= \Delta\frac{(1 - r)^2}{4c} - G \end{aligned}$$

We can see that the winner will end up making a loss whenever Δ turns out to be in the range:

$$\Delta \in [0, \frac{4cG}{(1-r)^2}]$$

A.0.2 The Auction

For simplicity of notation define the threshold value of Δ as: $\rho = \frac{4cG}{(1-r)^2}$. The entrant will bid in the auction such that: $Eu(\pi_{\max}) = 0$

$$\int_0^\rho (1 - e^{\lambda F(r)}) \frac{\Delta^m}{m!} e^{-\Delta} d\Delta + \int_\rho^\infty (1 - e^{-\lambda \frac{\Delta(1-r)^2}{4c}} e^{\lambda E} e^{\lambda F(r)}) \frac{\Delta^m}{m!} e^{-\Delta} d\Delta = 0$$

where the first integral on the LHS is his expected utility from winning the auction and then producing zero in the market, and the second integral is his expected utility from winning the auction and producing a positive amount in the market, and the RHS is what he gets if he loses. Note that the entry fee f is sunk at this stage.

$$\begin{aligned} & \int_0^\rho \frac{\Delta^m}{m!} e^{-\Delta} d\Delta - e^{\lambda F(r)} \int_0^\rho \frac{\Delta^m}{m!} e^{-\Delta} d\Delta + \\ & \int_\rho^\infty \frac{\Delta^m}{m!} e^{-\Delta} d\Delta - e^{\lambda E} e^{\lambda F(r)} \int_\rho^\infty e^{-\lambda \frac{\Delta(1-r)^2}{4c}} \frac{\Delta^m}{m!} e^{-\Delta} d\Delta = 0 \end{aligned}$$

Define:

$$x = \Delta(1 + \frac{\lambda(1-r)^2}{4c}) = \Delta\beta$$

which means the following hold:

$$\begin{aligned} dx &= \left(1 + \frac{\lambda(1-r)^2}{4c}\right) d\Delta \\ \Delta &= \rho \iff x = \rho\left(1 + \frac{\lambda(1-r)^2}{4c}\right) \\ \Delta &= 0 \iff x = 0 \\ \Delta &= \infty \iff x = \infty \end{aligned}$$

Substituting x in we get:

$$\begin{aligned} \int_0^\rho \frac{\Delta^m}{m!} e^{-\Delta} d\Delta + \int_\rho^\infty \frac{\Delta^m}{m!} e^{-\Delta} d\Delta - e^{\lambda F(r)} \int_0^\rho \frac{\Delta^m}{m!} e^{-\Delta} d\Delta \\ - e^{\lambda E} e^{\lambda F(r)} \frac{1}{\beta^{m+1}} \int_{\rho\beta}^\infty \frac{x^m}{m!} e^{-x} dx = 0 \end{aligned}$$

Note that the first two terms sum to one such that:

$$1 - e^{\lambda F(r)} \int_0^\rho \frac{\Delta^m}{m!} e^{-\Delta} d\Delta - e^{\lambda E} e^{\lambda F(r)} \frac{1}{\beta^{m+1}} \int_{\rho\beta}^\infty \frac{x^m}{m!} e^{-x} dx = 0$$

Define:

$$\int_\rho^\infty \frac{x^m}{m!} e^{-x} dx = \Phi(\rho)$$

Note:

1. $\Phi(\rho)$ is a function of the lower bound of a positive integral, it is therefore a decreasing function of ρ , and since ρ is increasing with r , $\Phi(\rho)$ is also a decreasing function of r .

2. Since x is a product of Δ and a constant we have:

$$\begin{aligned} \text{prob}(x \leq y) &= \text{prob}(\Delta k \leq y) = \text{prob}(\Delta \leq \frac{y}{k}) = \\ \frac{1}{m!} \int_0^{\frac{y}{k}} \Delta^m e^{-\Delta} d\Delta &= \frac{1}{m!} \int_0^y \left(\frac{x}{k}\right)^m e^{-\left(\frac{x}{k}\right)} \frac{dx}{k} = \text{prob}(\Delta \leq y) \end{aligned}$$

which means that x is also distributed Gamma (with the same parameter m). It therefore follows that:

$$\int_0^\rho \frac{\Delta^m}{m!} e^{-\Delta} d\Delta = 1 - \Phi(\rho)$$

3. From the definition of β and ρ it follows that:

$$\rho\beta = \rho + \lambda E$$

We therefore have:

$$\begin{aligned} 1 - e^{\lambda F(r)}(1 - \Phi(\rho)) - \frac{e^{\lambda E} e^{\lambda F(r)}}{\beta^{m+1}} \Phi(\rho + \lambda E) &= 0 \\ \frac{1 - e^{\lambda F(r)}}{e^{\lambda F(r)} \Phi(\rho)} + 1 &= \frac{e^{\lambda E}}{\beta^{m+1}} \left(\frac{\Phi(\rho + \lambda E)}{\Phi(\rho)} \right) \end{aligned}$$

Taking the logarithm of both sides we get:

$$\ln\left\{1 + \frac{1 - e^{\lambda F(r)}}{e^{\lambda F(r)} \Phi(\rho)}\right\} = \ln\left\{\frac{e^{\lambda E}}{\beta^{m+1}} \left(\frac{\Phi(\rho + \lambda E)}{\Phi(\rho)}\right)\right\}$$

Using a second order Taylor series approximation, assuming λ is positive but small, and neglecting terms in λ^3 , we get the following three:

$$\begin{aligned} \ln\left\{1 + \frac{1 - e^{\lambda F(r)}}{e^{\lambda F(r)} \Phi(\rho)}\right\} &= \frac{1 - e^{\lambda F(r)}}{e^{\lambda F(r)} \Phi(\rho)} - \frac{1}{2} \left(\frac{1 - e^{\lambda F(r)}}{e^{\lambda F(r)} \Phi(\rho)} \right)^2 \\ \ln\left\{1 + \frac{\lambda(1-r)^2}{4c}\right\}^{-(m+1)} &= -(m+1) \left(\frac{\lambda(1-r)^2}{4c} - \frac{1}{2} \frac{\lambda^2(1-r)^4}{16c^2} \right) \\ \ln \Phi(\rho + \lambda E) &= \ln \Phi(\rho) + \lambda E \frac{\frac{d\Phi}{d\rho}}{\Phi(\rho)} \end{aligned}$$

Using these we have:

$$\begin{aligned} \frac{1 - e^{\lambda F(r)}}{e^{\lambda F(r)} \Phi(\rho)} - \frac{1}{2} \left(\frac{1 - e^{\lambda F(r)}}{e^{\lambda F(r)} \Phi(\rho)} \right)^2 &= \lambda E - (m+1) \left(\frac{\lambda(1-r)^2}{4c} - \frac{1}{2} \frac{\lambda^2(1-r)^4}{16c^2} \right) + \\ &+ \ln \Phi(\rho) + \lambda E \frac{\frac{d\Phi}{d\rho}}{\Phi(\rho)} - \ln \Phi(\rho) \end{aligned}$$

Multiplying through by ρ :

$$\begin{aligned} \frac{1 - e^{\lambda F(r)}}{e^{\lambda F(r)}} - \frac{1}{2} \frac{(1 - e^{\lambda F(r)})^2}{e^{2\lambda F(r)} \Phi(\rho)} &= \\ \lambda E \Phi(\rho) - (m+1) \Phi(\rho) \left(\frac{\lambda(1-r)^2}{4c} - \frac{1}{2} \frac{\lambda^2(1-r)^4}{16c^2} \right) + \lambda E \frac{d\Phi}{d\rho} & \\ \frac{1}{e^{\lambda F(r)}} - 1 - (1 - e^{\lambda F(r)})^2 e^{-2\lambda F(r)} \frac{1}{2\Phi(\rho)} &= \\ \lambda E \Phi(\rho) + \lambda E \frac{d\Phi}{d\rho} - (m+1) \Phi(\rho) \left(\frac{\lambda(1-r)^2}{4c} - \frac{1}{2} \frac{\lambda^2(1-r)^4}{16c^2} \right) & \end{aligned}$$

For simplicity, we can work separately on each side of the equation. First the LHS:

$$\begin{aligned} \frac{1}{e^{\lambda F(r)}} - 1 - e^{-2\lambda F(r)} (1 - 2e^{\lambda F(r)} + e^{2\lambda F(r)}) \frac{1}{2\Phi(\rho)} & \\ e^{-\lambda F(r)} - 1 - \frac{1}{2\Phi(\rho)} (e^{-2\lambda F(r)} - 2e^{-\lambda F(r)} + 1) & \end{aligned}$$

Using a second order Taylor approximation again we get the following two:

$$\begin{aligned} e^{-\lambda F(r)} &= 1 - \lambda F(r) + \frac{1}{2} \lambda^2 (F(r))^2 \\ e^{-2\lambda F(r)} &= 1 - 2\lambda F(r) + 2\lambda^2 (F(r))^2 \end{aligned}$$

Still considering only the LHS we can substitute:

$$1 - \lambda F(r) + \frac{1}{2} \lambda^2 (F(r))^2 - 1 - \frac{1}{2\Phi(\rho)} (1 - 2\lambda F(r) + 2\lambda^2 (F(r))^2 - 2 + 2\lambda F(r) - \lambda^2 (F(r))^2 + 1)$$

Collecting terms and adding the RHS to the equation:

$$\begin{aligned} & \frac{1}{2}\lambda^2(F(r))^2 - \frac{1}{\Phi(\rho)}\frac{1}{2}\lambda^2(F(r))^2 - \lambda F(r) = \\ & \lambda E\Phi(\rho) + \lambda E\frac{d\Phi}{d\rho} - (m+1)\Phi(\rho)\left(\frac{\lambda(1-r)^2}{4c} - \frac{\lambda^2(1-r)^4}{32c^2}\right) \\ & (m+1)\Phi(\rho)\left(\frac{(1-r)^2}{4c} - \frac{\lambda(1-r)^4}{32c^2}\right) = \\ & E\Phi(\rho) + E\frac{d\Phi}{d\rho} + F(r) + \lambda(F(r))^2\left(\frac{1-\Phi(\rho)}{2\Phi(\rho)}\right) \end{aligned}$$

We can now formulate the entrant's bidding function:

$$E\left\{\Phi(\rho) + \frac{d\Phi}{d\rho}\right\} + F(r_E)\left\{1 + \lambda F(r_E)\frac{1-\Phi(\rho)}{2\Phi(\rho)}\right\} = \Phi(\rho)(m+1)\left\{\frac{(1-r)^2}{4c} - \frac{\lambda(1-r)^4}{32c^2}\right\} \quad (A1)$$

Setting $\lambda = 0$ we can construct the incumbent's bidding function:

$$I\left\{\Phi(\rho) + \frac{d\Phi}{d\rho}\right\} + F(r_I) = \Phi(\rho)\frac{(m+1)(1-r)^2}{4c} \quad (A2)$$

Comparing equations (A1) with (2) and (A2) with (3), and recalling that $\Phi(\rho) < 1$ for all $\rho > 0$, and that $\Phi(\rho) = -\frac{\rho^m}{m!}e^{-\rho} < 0$, it can be established that for a given set of costs I and E and a given fixed-fee function $F(r)$ both bidders will bid lower in this case relative to the case where they are sure to make a non-negative profit in the market. The main features of the bidders behaviour, however, seem to carry over to this more realistic case, at the cost of substantially complicating the mathematics. The possibility of a loss in the market therefore remains as an appendix to the main body

of this study, where the simplified, more tractable model assuming market demand always covers the costs, forms its body.

Bibliography

- [1] Abbink, K., Irlenbusch, B., Pezanis-Chriktou, P., Rockenbach, B., Sadrieh, A. & Selten, R., (2001), "An Experimental Test of Design Alternatives for the British 3G/UMTS Auction" Working Paper, University of Bonn.
- [2] Amemiya, T., (1994) *Introduction to Statistics and Econometrics*, Harvard University Press.
- [3] Athey, S., Levin, J. & Seira, E., (2004), "Comparing Open and Sealed Bid Auctions: Theory and Evidence from Timber auctions", Working paper Stanford University.
- [4] Bajari, P. & Hortacsu, A., (2000), "Winner's Curse, Reserve Prices and Endogenous Entry: Empirical Insights from eBay Auctions" Working paper Stanford University.
- [5] Binmore, K. & Klemperer, P., (2002), "The Biggest Auction Ever: The Sale Of The British 3G Telecom Licences", *The Economic Journal*, 112, pp. C74-C96.
- [6] Borgers, T. & Dustmann, C., (2003), "Awarding Telecom Licences: The Recent European Experience", *Economic Policy*, 36, pp. 217-268.
- [7] Bousquet, A., Cremer, H., Ivaldi, M., & Wolkowicz, M., (1998), "Risk Sharing in Licensing", *International Journal of Industrial Organization*, 16, pp. 535-554.
- [8] Cantillon, E., (2000), "The Effect of Bidders' Asymmetries on Expected Revenue in Auctions", Working Paper.
- [9] Cox, J. C., Smith, V. L. & Walker, J. M., (1985), "Experimental Development of Sealed-Bid Auction Theory: Calibrating Controls for Risk Aversion", *American Economic Review*, 75 (2), pp.160-165.
- [10] Cramton, P., (2001), "Spectrum Auctions", in the *Handbook of Telecommunications Economics*, Cave, M., Majumdar, S., & Vogelsang I., eds., Amsterdam Press.

- [11] Engelbrecht-Wiggans, R., (1993), "Optimal Auctions Revisited", *Games and Economic Behavior*, 5, pp. 227-239.
- [12] Fibich, G., Gaviious, A., & Sela, A., (2004), "All-Pay Auctions with Weakly Risk-Averse Buyers", Discussion Paper, Di Lavoro.
- [13] Goeree, J.K., Offerman, T. & Schram, A., (2003), "Using First-Price Auctions to Sell Heterogeneous Licences" Working Paper.
- [14] Guth, W., Ivanova-Stenzel, R. & Wolfstetter, E., (2001), "Bidding Behavior in Asymmetric Auctions: An Experimental Study" Working Paper Humboldt-University of Berlin.
- [15] Hansen, R.G., (1985), "Auctions with Contingent Payments", *The American Economic Review*, 75 (4), pp. 862-865.
- [16] Holt, C. A., (1980), "Competitive Bidding for Contracts under Alternative Auction Procedures", *Journal of Political Economy*, 88 (3), pp.433-445.
- [17] "Hong Kong Third Generation Mobile Services Licensing", Information Memorandum, July 2001, OFTA.
- [18] Ivanova-Stenzel, R. & Salmon, T., (2003), "Bidder Preferences among Auction Institutions" Working Paper Humboldt-University of Berlin.
- [19] Jehiel, P., & Moldovanu, B., (2001), "The European UMTS/IMT-2000 Licence Auctions", Working Paper University College London.
- [20] Kaplan, T., & Sela, A., (2003), "Auctions with Private Entry Costs", Discussion Paper, CEPR.
- [21] Kaplan, T. R., & Zamir, S., (2002), "A Note on Revenue Effects of Asymmetry in Private-Value Auctions", Working Paper The Hebrew University Israel.
- [22] Klemperer, P., (2001), "What Really Matters In Auction Design?", *Journal of Economic Perspectives*, 16 (1).
- [23] Krishna, V., (2002), *Auction Theory* Academic Press.

- [24] Leland, H. E., (1978), "Optimal Risk Sharing and the Leasing of Natural Resources, with Applications to Oil and Gas Leasing on the OCS", *The Quarterly Journal of Economics*, August 1978, pp. 413-437.
- [25] Levin, D., & Smith, J. L., (1994), "Equilibrium in Auctions with Entry", *The American Economic Review*, 84 (3), pp. 585-599.
- [26] Marshall, R. C., Meurer, M. J., Richard, J-F., Stromquist, W., (1994), "Numerical Analysis of First Price Auctions", *Games and Economic Behavior*, 7, pp. 193-220.
- [27] Maskin, E., & Riley, J., (1981), "The Gains to Making Losers Pay in High Bid Auctions" Discussion Paper #198 MIT.
- [28] Maskin, E., & Riley, J., (1984), "Optimal Auctions with Risk Averse Buyers", *Econometrica*, 52 (6), pp. 1473-1518.
- [29] ——— & ——— (2000), "Asymmetric Auctions", *The Review of Economic Studies*.
- [30] Matthews, S., (1987), "Comparing Auctions for Risk Averse Buyers: A Buyer's Point of View", *Econometrica*, 55 (3), pp. 633-646.
- [31] McAfee, P. R., & McMillan, J., (1996), "Analysing the airwaves Auction", *Journal of Economic Perspectives*, 10 (1), pp. 159-175.
- [32] McAfee, R. P., & McMillan, J., (1986), "Bidding for Contracts: A Principal-Agent Analysis" *Rand Journal of Economics*, 17 (3), pp. 326-338.
- [33] ——— & ——— (1987a), "Auctions with Entry" *Economic Letters*, 23, pp. 343-347.
- [34] ——— & ——— (1987b), "Auctions and Bidding", *Journal of Economic Literature*, XXV, pp. 699-738.
- [35] McMillan, J., (1994), "Selling Spectrum Rights", *Journal of Economic Perspectives*, 8 (3), pp. 145-162.
- [36] Milgrom, P. R., & Weber, R. J., (1982), "A Theory of Auctions and Competitive Bidding", *Econometrica*, 50 (5), pp. 1089-1122.

- [37] Moody, C. E., (1994), "Alternative Bidding Systems for Leasing Offshore Oil: Experimental Evidence", *Econometrica*, 61, pp. 345-353.
- [38] Moore, J., (1984), "Global Incentive Contracts in Auction Design" *Econometrica*, 52 (6), pp. 1523-1535.
- [39] Palfrey, T. & Pevnitskaya, S., (2003), "Endogenous Entry and Self-Selection in Private Value Auctions: An Experimental Study", Working Paper, California Institute of Technology.
- [40] Reece, D. K., (1979), "An Analysis of Alternative Bidding Systems for Leasing Offshore Oil", *The Bell Journal of Economics*, pp. 659-669.
- [41] Riley, J. G., (1988), "Ex Post Information in Auctions", *Review of Economic Studies*, pp. 409-430.
- [42] ——— (1996), "Characterizing Asymmetric Auctions", Working Paper.
- [43] Riley, J. G., & Samuelson, W. F., (1981), "Optimal Auctions", *The American Economic Review*, 71 (3), pp. 381-392.
- [44] Roth, A. E. & Malouf W. K., (1979), "Game Theoretic Models and the Role of Information in Bargaining: An Experimental Study" *Psychological Review*, 86, pp. 574-594.
- [45] Swierzbinski, J., "Preliminary Notes on Asymmetric Auction Experiment", August 2003.
- [46] Vickrey, W., (1961), "Counterspeculation, Auctions, and Competitive Sealed Tenders", *The Journal of Finance*, 16 (1), pp. 8-37.